

The Design and Pricing of Modern Structured Products – A Note[§]

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ABSTRACT

In this paper, we show that in the design of modern structured products, the initial price of the underlying asset vanishes in the profit function of the issuer. The independence of the issuer's profit function of the underlying asset price provides flexibility to the issuing institution. Once designed, a structured product can be issued any time before maturity, regardless of the price of the underlying asset. We feature a yield-enhancing product, reverse exchangeable bond, and further support our conclusion through two additional types of modern structured products, a capital protection product (Equity-Linked CDs), and a participation product (Outperformance Certificates).

JEL classification: G13, G24

Keywords: Structured Products; Reverse Exchangeable Bond; Discount Certificate; Knock-In; Knock-Out; Equity-Linked CD; Outperformance Certificate;

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1. INTRODUCTION

The origin of modern structured products dates back to the early 1990s, in the United States, with the creation of Equity-Linked Certificates of Deposit (i.e., ELCD), also known as capital protection products. There is no simple and generally accepted classification of modern structured products. In 2010, the Swiss Structured Product Association (SSPA) proposed and published a classification called the Swiss Derivative Map of structured products. Following the SSPA classification, modern structured products can be classified, in general terms, into four primary groups.

The first group, capital protection products, rose in popularity because they provided capital protection while at the same time afforded participation in the gain of the underlying asset over the term to maturity. Banks created further modifications of the equity-linked certificates of deposit wherein capital protection is not guaranteed; however, there is upside participation. This second group is known as participation products. Within this group, we can identify two subgroups being Outperformance Certificates (i.e., OC), providing no capital protection and Bonus Certificates (i.e., BC), which provides capital protection as long as the underlying asset does not drop below a lower barrier. Yield enhancing products, the third group, have payoffs where investors participate in the losses and gains of the underlying assets, albeit the gains may be limited and even nonexistent at times. Within this group, we can identify four subgroups being

Discount Certificates (i.e., DCs), Reverse Exchangeable Bonds (i.e., REXs), Capped Outperformance Certificates, and Capped Bonus Certificates. The fourth group, known as leverage products, consists of a group of securities that replicate call options, put options or combinations thereof. Within the category, we can identify three subgroups being Warrants, Spread Warrants, and Knock-Out Warrants.

In this paper, we show that in the design of modern structured products, the initial price of the underlying asset vanishes in the profit function of the issuer. The independence of the issuer's profit function of the underlying asset price provides flexibility to the issuing institution. Once designed, a structured product can be issued any time before maturity, regardless of the price of the underlying asset. We feature a yield-enhancing product, a Reverse Exchangeable Bond (i.e., a REX), and further supporting our conclusion; we present two additional types of modern structured products, a capital protection product (i.e., an ELCD) and a participation product (i.e., an OC).

2. DESIGN AND PRICING OF MODERN STRUCTURED PRODUCTS

In this section, we show how the initial price or value of the underlying asset of three modern structured products being reverse exchangeable bonds (i.e., REX), equity-linked certificates of deposit (i.e., ELCD), and outperformance certificates (i.e., OC) disappears from the issuers' profit function. We further exhibit issuances of these modern structured

products can take place before the term to maturity since the issuers' profit function is unaffected by the initial price or value of the underlying asset.

2.1 REVERSE EXCHANGEABLE BONDS

Reverse exchangeable securities are one of the most popular *modern structured products*, experiencing explosive growth during the last two decades (Hernandez et al., 2010). A plain vanilla *reverse exchangeable bond* is a bond that pays a principal payment on maturity date contingent upon the price of a pre-specified asset (to be referred as the *underlying asset*) on the maturity date of the bond. If the closing price of the underlying asset on the bond maturity date is *equal to* or *higher than* the pre-determined price (to be referred to as the *exercise price* or *strike price*), the bond investors will receive the “face value” of the bond (usually \$1,000) from the bond issuer. However, if the closing price of the underlying asset is *below* the *exercise price*, the investors will receive a fixed number of shares of the underlying asset. Usually, the exercise price is set equal to the price of the underlying asset on the bond issue date (to be referred to as the initial asset price or *initial price*). In Appendix 1, we present an example of a reverse exchangeable bond.

$$V_T = \begin{cases} \$1,000 & \text{if } S_T \geq S_0 \\ S_T \frac{\$1,000}{S_0} & \text{if } S_T < S_0 \end{cases} \quad \dots(1)$$

When investors purchase a reverse exchangeable bond, they basically engage in the following transactions simultaneously, either¹: 1) they take a *long* position in a fixed-rate

¹ This relationship can be seen easily from the put-call parity

$$Xe^{-rt} - P = S - C$$

bond, and they *short* several contracts of *put* options, or 2) they take *long* positions in zero-coupon bonds, they take a *long* position in the underlying asset, and they *short* several contracts of call options. The *underlying asset* of the put (call) option is the underlying asset of the reverse exchangeable, the *exercise price* of the put (call) option is the *initial price* of the underlying asset, the *expiration date* of the put (call) option is the maturity date of the reverse exchangeable, and the *number of contracts* written is the face value of the reverse exchangeable bond (usually \$1,000) divided by the initial price of the underlying asset. The high coupon payments made by the reverse exchangeable basically include the option premium paid by the bond issuer to the investors of the reverse exchangeable bonds. The relationship between the redemption value on a reverse exchangeable bond and the replication strategies can be represented in Figure 1 and Figure 2.

Since the payoff of a plain vanilla reverse exchangeable bond is the same as the payoff of the replicating strategies, we can price the fair value of the reverse exchangeable based on the total cost of any replicating strategy (Mason, 1995). Therefore, any selling price of the bonds above the fair value is the gain to the bond issuer.² Following the first replicating strategy, the profit function, Π for the issuing firm is:

$$\Pi = B_0 - \sum_i^n C e^{-r t_i} - V_T$$

² For studies on the pricing of reverse convertible bonds, see Benet et al., 2006; Burth et al., 2001; Hernandez et al., 2010; Stoimenov and Wilkens, 2005; and Wilkens et al., 2003.

$$= B_0 - \sum_i^n C e^{-r t_i} - \$1,000 e^{-r T} + \frac{\$1,000}{S_0} P \quad \dots(2)$$

The profit Π function for the issuer of a reverse exchangeable bond has four components: (1) the bond price received by the issuer (B_0); (2) the present value of all coupon payments promised by the issuer ($\sum_i^n C_i e^{-r t_i}$); (3) the present value of the bond's face value promised by the issuer ($\$1,000 e^{-r T}$); and finally (4) the value of $\frac{\$1,000}{S_0}$ shares of put options with each option having the value P^3 :

$$P = S_0 e^{-r T} N(-d_2) - S_0 e^{-q T} N(-d_1) \quad \dots(3)$$

Where r is the risk-free rate of interest, q is the dividend yield of the underlying assets, T is the term to maturity of the reverse exchangeable bond, $X (\equiv S_0)$ is the exercise price and

$$d_1 = \frac{\ln\left(\frac{S_0}{S_0}\right) + \left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad \dots(4)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad \dots(5)$$

Where σ is the standard deviation of the underlying asset return.

$$\begin{aligned} \Pi &= B_0 - \sum_i^n C e^{-r t_i} - \$1,000 e^{-r T} + \frac{\$1,000}{S_0} [S_0 e^{-r T} N(-d_2) - S_0 e^{-q T} N(-d_1)] \\ &= B_0 - \sum_i^n C_i e^{-r t_i} - \$1,000 e^{-r T} + \$1,000 [e^{-r T} N(-d_2) - e^{-q T} N(-d_1)] \quad \dots(6) \end{aligned}$$

³ The pricing formula for this put option is a special case of the Black-Scholes general model for a put in that the exercise price, X , is the same as the initial underlying asset price (i.e., $X = I_0$). See Black and Scholes, 1973.

It is worth noting that, although the initial price S_0 is explicitly specified in the contract of reverse exchangeable bonds, it turns out that S_0 neither reflected in the profit function for issuing such a bond (Equation (6)) nor in the value of the put option embedded in the bond (the last term in Equation (6)). Instead, S_0 vanishes both in Equation (6) and in d_1 and d_2 . The fact that both the profit function for issuing a reverse exchangeable bond and the embedded put option in the bond are *independent* of the initial price S_0 is a very important feature in the design of a reverse exchangeable because once a reverse exchangeable bond is designed, it can be issued *any time* before maturity regardless the price of the underlying asset since the issuer's profit will not be affected by the price.

In addition to the plain vanilla reverse exchangeable, there are three other types of reverse exchangeable bonds; they are discount certificates, *knock-in* reverse exchangeable bonds, and *knock-out* reverse exchangeable bonds. The same conclusions apply to all four types of reverse exchangeable bonds (See Appendix 2 for greater details).

2.2 EQUITY LINKED CERTIFICATES OF DEPOSIT

Equity Linked Certificates of Deposit (ELCD) were the first wave of modern structured products created. ELCDs, in their simplest version, provide full capital protection and 100% upside participation in the gains of the underlying asset. Assume an initial value of \$1 is invested in an ELCD at $t = 0$ when the initial index is I_0 . The ELCD provides 100 percent capital protection and a participation rate of 100 percent in the positive return of

the underlying index, then the *terminal value* of the investment at $t = T$, V_T , can be expressed mathematically as:

$$V_T = \begin{cases} \$1 & \text{if } I_T \leq I_0 \\ \frac{\$1}{I_0} I_T & \text{if } I_T > I_0 \end{cases}$$

$$= \$1 + \frac{1}{I_0} \text{Max} [0, I_T - I_0] \quad \dots(7)$$

The payoff expressed in Equation (7) can be duplicated by the combination of a zero-coupon bond and a call option.⁴ The payoff of \$ 1 can be duplicated by a zero-coupon bond with a face value of \$1 (which costs e^{-rT} at $t = 0$). The second term of Equation (7) can be duplicated by $1/I_0$ of a call, C , that has an exercise price of I_0 . The cost of the call option, C , based on the Black-Scholes European option pricing model, is

$$C = I_0 e^{-qT} N(d_1) - I_0 e^{-rT} N(d_2) \quad \dots(8)$$

Where I_0 is the current underlying asset price, T is the term to expiration of the option, r is the risk-free rate of interest, and q is the dividend yield of the underlying assets. $N(d_1)$ and $N(d_2)$ are the cumulative probabilities for standardized normal distribution of d_1 and d_2 , respectively, and σ is the volatility of the underlying index.

$$d_1 = \frac{\left(r - q + \frac{1}{2} \sigma^2 \right) T}{\sigma \sqrt{T}} \quad \dots(9)$$

$$d_2 = d_1 - \sigma \sqrt{T} \quad \dots(10)$$

⁴ Alternatively, the payoff expressed in Equation (7) can be duplicated by a long position in the index itself and a long position in a put option on the index based on the put-call parity for European options:

$$C + X e^{-rT} = S + P$$

Where C and P are the premiums for of call and put options with a term of expiration of T and an exercise price of X on an underlying asset price of S when the risk-free rate of interest is r . The right-hand side of the Equation is a combination of a call and a zero-coupon bond, while the left-hand side of the Equation is a combination of a put and an underlying asset.

When the issuer of ELCDs charges B_0 for the ELCD, the issuer's profit, π , is

$$\pi = B_0 - e^{-rT} - \frac{\$1}{I_0} C \quad \dots(11)$$

$$\begin{aligned} \pi &= B_0 - e^{-rT} - [e^{-qT} N(d_1) - e^{-rT} N(d_2)] \\ &= (B_0 - e^{-qT}) - [e^{-rT} [1 - N(d_2)] - e^{-qT} [1 - N(d_1)]] \end{aligned} \quad \dots(12)$$

It turns out that the profit for issuing the ELCDs is not affected by the initial index level I_0 since I_0 vanishes in Equations (12).

2.3 OUTPERFORMANCE CERTIFICATES

The rate of return of a certificate is contingent upon the price performance of its underlying asset over its term to maturity. If we denote I_0 as the underlying asset price on the fixing date, I_T as the *valuation price*, PF as the *performance factor*, then for an initial investment of \$1 in one *uncapped* certificate, the total value that an investor will receive on the *expiration date* (known as the *redemption value* or *settlement amount*), V_T , is equal to:

$$V_T = \$1/I_0 \begin{cases} I_0 \times \left[1 + PF \frac{(I_T - I_0)}{I_0} \right] & \text{if } I_T > I_0 \\ I_0 \times \left[1 + \frac{(I_T - I_0)}{I_0} \right] & \text{if } I_T \leq I_0 \end{cases} \quad \dots(13)$$

If we define APF (*additional performance factor*) as performance factor minus one, i.e., $APF \equiv (PF - 1)$, then

$$\begin{aligned} V_T &= \$1/I_0 \begin{cases} I_0 + (1 + APF)(I_T - I_0) & \text{if } I_T > I_0 \\ I_T & \text{if } I_T \leq I_0 \end{cases} \\ &= \$1/I_0 (I_T + \text{Max}[0; APF (I_T - I_0)]) \end{aligned}$$

$$= \$1/I_0 (I_T + APF \times \text{Max}[0; I_T - I_0]) \quad \dots(14)$$

A long position in the underlying asset will generate a payoff I_T on maturity date T *plus* cash dividends on ex-dividend dates. Since Outperformance Certificates do *not* pay cash dividends, the payoff I_T in Equation (14) can be duplicated by taking a *long* position on the underlying asset, and a *short* position on the zero-coupon bond of which the face values are equal to the amount of dividends and the maturity dates are the ex-dividend dates. The payoff $\text{Max}[0, I_T - I_0]$ in Equation (14) is the payoff of a long position for a call on the underlying asset with an exercise price I_0 . So, the payoff for investing in one Outperformance Certificate is the same as the combined payoffs of taking the following three positions: (1) A long position in the underlying asset; (2) A short position in zero-coupon bonds. The face values of the bonds are the cash dividends to be paid by the underlying asset, and the maturity dates are the ex-dividend dates of cash dividends; (3) A long position in call options on the underlying asset. The number of options is the performance factor minus one (known as additional performance factor). The exercise price of the options is I_0 , and the term to expiration of the options is T , the same as the term to maturity of the certificate.

Since the payoff of uncapped certificates is the same as the combined payoffs of the above three positions, we can calculate the fair value of the certificates based on the value of the three positions. Any selling price of the certificates above the value of the above three positions is the gain to the certificate issuer.

The value of *Position 1* is the price of the underlying asset on fixing date I_0 .

The value of *Position 2* is the present value of cash dividends to be paid by the underlying asset, to be denoted as PV_D . The value of *Position 3* is the value of APF shares of call options with each call value of C where

$$C = I_0 e^{-qT} N(d_1) - I_0 e^{-rT} N(d_2) \quad \dots(15)$$

Where r is the risk-free rate of interest, q is the dividend yield of the underlying assets, T is the term to maturity of the certificate, X is the exercise price and

$$d_1 = \frac{\ln\left(\frac{I_0}{X}\right) + \left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad \dots(16)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad \dots(17)$$

Where σ is the standard deviation of the underlying asset return. Therefore, the total cost, TC , for each uncapped certificate is

$$TC = \$1/I_0 [I_0 - PV_D + APF [I_0 e^{-qT} N(d_1) - I_0 e^{-rT} N(d_2)]] \quad \dots(18)$$

And the profit function for the issuer is

$$\begin{aligned} \Pi &= P - TC \\ \Pi &= P - \$1/I_0 [I_0 - PV_D + APF [I_0 e^{-qT} N(d_1) - I_0 e^{-rT} N(d_2)]] \end{aligned} \quad \dots(19)$$

Where P is the issue price of one uncapped certificate.

Equations (18) and (19) are based on continuous dividend yield. Since the dividends for individual stocks are discrete, we use the following approach to calculate the equivalent

continuous dividend yield for stocks that pay discrete dividends. For an underlying asset which is an individual stock with a price I_0 at $t=0$ (the issue date) and which pays n dividends during a time period T with cash dividend D_i being paid at time t_i , the equivalent dividend yield q will be such that

$$I_0 - \sum_{i=1}^n D_i e^{-r t_i} = I_0 e^{-qT} \quad \dots(20)$$

$$e^{-qT} = \frac{I_0 - \sum_{i=1}^n D_i e^{-r t_i}}{I_0} = 1 - \frac{\sum_{i=1}^n D_i e^{-r t_i}}{I_0} \quad \dots(21)$$

$$-qT = \ln \left[1 - \frac{\sum_{i=1}^n D_i e^{-r t_i}}{I_0} \right] \quad \dots(22)$$

$$q = - \frac{\ln \left[1 - \frac{\sum_{i=1}^n D_i e^{-r t_i}}{I_0} \right]}{T} \quad \dots(23)$$

$$\Pi = P - \$1/I_0 [I_0 e^{-qT} + APF [I_0 e^{-qT} N(d_1) - I_0 e^{-rT} N(d_2)]] \quad \dots(24)$$

$$\Pi = P - [e^{-qT} + APF [e^{-qT} N(d_1) - e^{-rT} N(d_2)]] \quad \dots(25)$$

It turns out that the profit for issuing the Outperformance Certificate is not affected by the initial index level I_0 since I_0 vanishes in Equations (25).

3. CONCLUSION

In this paper, we show that in the design of modern structured products, the initial price of the underlying asset, explicitly specified in the contract of the certificates, vanishes in the profit function of the issuer. The independence of the issuer's profit function of the underlying asset price provides great flexibility to the issuing institution.

Once the structured product is designed, it can be issued any time before maturity regardless of the price of the underlying asset price. We show those same conclusions apply to four types of Reverse Exchangeable Bonds (i.e., plain vanilla reverse exchangeable bonds, discount certificates, knock-in reverse exchangeable bonds, and knock-out reverse exchangeable bonds), an Outperformance Certificate, and an Equity Linked Certificate of Deposit.

REFERENCES

- Benet, B., Giannetti, A. and Pissaris, S., 2006. Gains from structured product markets: The case of reverse-Convertible securities (RES). *Journal of Banking and Finance* 30, 111--132.
- Black, F. and Scholes, M., 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81, 637--659.
- Burth, S., Kraus, T. and Wohlwend, H., 2001. The pricing of structured products in the Swiss market. *Journal of Derivatives* 9 (Winter), 30--40.
- Hernandez, R., Lee, W., and Liu, P., 2010. An Economic Analysis of Reverse Convertible Securities — An Option-Pricing Approach. *Review of Futures Markets* 19, 1, 67--95.
- Hull, John, 2003. *Options, Futures, and Other Derivatives*, Fifth Edition, Pearson Education, Inc. Upper Saddle River, New Jersey.
- Mason, S., 1995. Contingent Claim Analysis, in: Mason, S., Merton, R, Perold, A., and Tufano, P., *Cases in Financial Engineering: Applied Studies of Financial Innovation*, Prentice Hall, Inc. Englewood Cliffs, New Jersey.
- Stoimenov, P., and Wilkens, S., 2005. Are structured products ‘fairly’ priced? An analysis of the German market for equity-linked instruments. *Journal of Banking and Finance* 29, 2971--2993.
- Wilkens, S., Erner, C., and Roder, K., 2003. The pricing of structured products in German market. *Journal of Derivatives* 11 (Fall), 55--69.

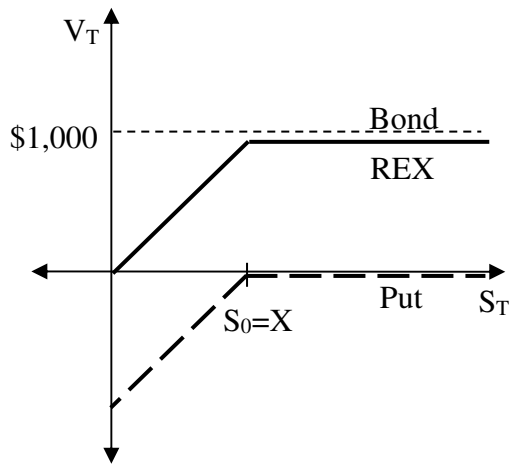


Figure 1: The terminal payoff of a reverse exchangeable bond, V_T , a bond, and a short position in put options as a function of the price of the underlying asset, S_T .

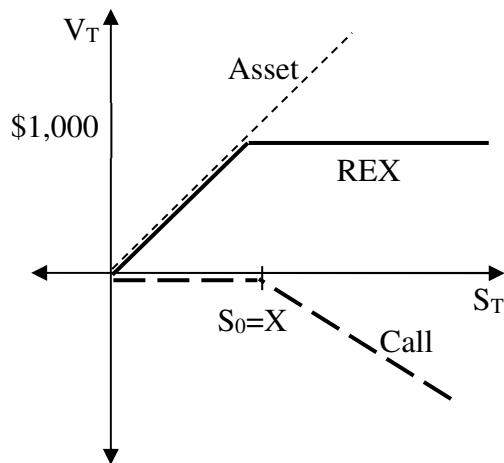


Figure 2: The terminal payoff of a reverse exchangeable bond, V_T , underlying asset, and a short position in call options as a function of the price of the underlying asset, S_T .

Appendix 1

PRICING SUPPLEMENT
(TO PROSPECTUS DATED SEPTEMBER 17, 2003
AND PROSPECTUS SUPPLEMENT
DATED SEPTEMBER 18, 2003)

Pricing Supplement No. 43 to
Registration Statement No. 333-89136
Dated February 13, 2004
Rule 424(b)(3)



\$8,250,000
ABN AMRO Bank N.V.
MEDIUM-TERM NOTES, SERIES A
Senior Fixed Rate Notes

fully and unconditionally guaranteed by
ABN AMRO Holding N.V.

10.00% Reverse Exchangeable Securities due February 18, 2005
linked to common stock of Texas Instruments Incorporated

The Securities do not guarantee any return of principal at maturity. Instead, if the closing price of the shares of common stock of Texas Instruments Incorporated, which we refer to as the Underlying Shares, is below a certain level on the third trading day prior to the maturity date, which we refer to as the determination date, we will exchange each Security for a predetermined number of Underlying Shares. The market value of those shares will be less than the principal amount of each Security and could be zero.

Securities	10.00% Reverse Exchangeable Securities due February 18, 2005.	Payment at Maturity	The payment at maturity is based on the closing price of the Underlying Shares on the determination date.
Principal Amount	\$8,250,000		• If the closing price per Underlying Share on the determination date is at or above the initial price, we will pay the principal amount of each Security in cash.
Underlying Shares	Common stock, par value \$1.00 per share of Texas Instruments Incorporated.		• If the closing price per Underlying Share on the determination date is below the initial price, we will deliver to you, in exchange for each \$1,000 principal amount of the Securities, a number of Underlying Shares equal to the stock redemption amount.
Interest Rate	10.00% per annum, payable semi-annually in arrear on August 19, 2004 and February 18, 2005.		• You will receive cash in lieu of fractional shares.
Issue Price	100%		
Original Issue Date (Settlement Date)	February 19, 2004		
Maturity Date	February 18, 2005		
Initial Price	\$30.77 (the closing price per Underlying Share on February 13, 2004, the date we priced the Securities, subject to adjustment for certain corporate events affecting the Underlying Shares, which we describe in "Description of Securities — Adjustment Events").	Guarantee	The Securities will be fully and unconditionally guaranteed by ABN AMRO Holding N.V.
		Denominations	The Securities may be purchased in denominations of \$1,000 and integral multiples thereof.
Stock Redemption Amount	32.499 Underlying Shares for each \$1,000 principal amount of the Securities, which is equal to \$1,000 divided by the initial price.	No Affiliation with Texas Instruments Incorporated	Texas Instruments Incorporated, which we refer to as Texas Instruments, is not an affiliate of ours and is not involved with this offering in any way. The obligations represented by the Securities are our obligations, not those of Texas Instruments. Investing in the Securities is not equivalent to investing in Texas Instruments common stock.
Determination Date	The third trading day prior to the maturity date, subject to adjustment in certain circumstances which we describe in "Description of the Securities — Determination Date".	Listing	We do not intend to list the Securities on any securities exchange.

The Securities are not insured by the Federal Deposit Insurance Corporation or any other federal agency. The Securities involve risks not associated with an investment in conventional debt securities. See "Risk Factors" beginning on PS-7.

The Securities and Exchange Commission and state securities regulators have not approved or disapproved these Securities, or determined if this Pricing Supplement or the accompanying Prospectus or Prospectus Supplement is truthful or complete. Any representation to the contrary is a criminal offense. The agents are not obligated to purchase the Securities but have agreed to use reasonable efforts to solicit offers to purchase the Securities. The total aggregate principal amount of the Securities being offered by this Pricing Supplement was not purchased by investors in the offering. One or more of our affiliates has agreed to purchase the unsold portion, which does not exceed \$825,000 and to hold such Securities for investment for a period of at least 30 days. See "Holding of the Securities by Our Affiliates and Future Sales" under the heading "Risk Factors" and "Plan of Distribution."

This Pricing Supplement and the accompanying Prospectus Supplement and Prospectus may be used by our affiliates in connection with offers and sales of the Securities in market-making transactions.

PRICE \$1,000 PER SECURITY
ABN AMRO Financial Services, Inc.
ABN AMRO Incorporated

February 13, 2004

Appendix 2

Discount Certificates

$$\begin{aligned}
 \Pi &= B_0 - V_T \\
 &= B_0 - \$1,000e^{-rT} + \frac{\$1,000}{I_0} P; & P &= I_0e^{-rT} N(-d_2) - I_0e^{-qT} N(-d_1) \\
 &= B_0 - \$1,000e^{-rT} + \$1,000 \left[e^{-rT} N(-d_2) - e^{-qT} N(-d_1) \right]
 \end{aligned}$$

Where,

$$\begin{aligned}
 d_1 &= \frac{\left(r - q + \frac{1}{2} \sigma^2 \right) T}{\sigma \sqrt{T}} \\
 d_2 &= d_1 - \sigma \sqrt{T}
 \end{aligned}$$

Knock-In Reverse Convertibles

$$\begin{aligned}
 \Pi &= B_0 - \sum_i^n C_i e^{-r t_i} - \$1,000 e^{-rT} + \frac{\$1,000}{I_0} P_{di} \\
 P_{di} &= -I_0 e^{-qT} N(-x_1) + I_0 e^{-rT} N(-x_1 + \sigma \sqrt{T}) + I_0 e^{-qT} (H')^{2\lambda} [N(y) - N(y_1)] \\
 &\quad - I_0 e^{-rT} (H')^{2\lambda-2} [N(y - \sigma \sqrt{T}) - N(y_1 - \sigma \sqrt{T})]
 \end{aligned}$$

Where H' is the knock-in level as a percentage of the exercise price, and

$$\begin{aligned}
 x_1 &= \frac{\ln\left(\frac{1}{H'}\right)}{\sigma \sqrt{T}} + \lambda \sigma \sqrt{T} & y &= \frac{\ln(H'^2)}{\sigma \sqrt{T}} + \lambda \sigma \sqrt{T} \\
 y_1 &= \frac{\ln(H')}{\sigma \sqrt{T}} + \lambda \sigma \sqrt{T} & \lambda &= \frac{(r - q) + \frac{\sigma^2}{2}}{\sigma^2}
 \end{aligned}$$

$$\begin{aligned}
 \Pi &= B_0 - \sum_i^n C_i e^{-r t_i} - \$1,000 e^{-rT} \\
 &\quad + \$1,000 \left(-e^{-qT} N(-x_1) + e^{-rT} N(-x_1 + \sigma \sqrt{T}) + e^{-qT} (H')^{2\lambda} [N(y) - N(y_1)] \right. \\
 &\quad \left. - e^{-rT} (H')^{2\lambda-2} [N(y - \sigma \sqrt{T}) - N(y_1 - \sigma \sqrt{T})] \right)
 \end{aligned}$$

Knock-Out Reverse Convertibles

$$\Pi = B_0 - \sum_i^n C_i e^{-r t_i} - \$1,000 e^{-rT} + \frac{\$1,000}{I_0} P_{uo}$$

$$P_{uo} = P - P_{ui}$$

where

P: is the regular put premium

P_{ui} : is the premium for the up-and-in put and

$$P_{ui} = -I_0 e^{-qT} (H')^{2\lambda} N(-y) + I_0 e^{-rT} (H')^{2\lambda-2} N(-y + \sigma\sqrt{T})$$

Where H' is the knock-out level as a percentage of the exercise price, and

$$\lambda = \frac{(r-q) + \frac{\sigma^2}{2}}{\sigma^2}$$

$$\Pi = B_0 - \sum_i^n C_i e^{-r t_i} - \$1,000 e^{-rT} + \frac{\$1,000}{I_0} [P - P_{ui}]$$

$$\begin{aligned} \Pi = B_0 - \sum_i^n C_i e^{-r t_i} - \$1,000 e^{-rT} \\ + \$1,000 \{ [e^{-rT} N(-d_2) - e^{-qT} N(-d_1)] \\ - [-e^{-qT} (H')^{2\lambda} N(-y) + e^{-rT} (H')^{2\lambda-2} N(-y + \sigma\sqrt{T})] \} \end{aligned}$$

$$y = \frac{\ln(H'^2)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$\lambda = \frac{(r-q) + \frac{\sigma^2}{2}}{\sigma^2}$$