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"GROSS" PRESENT VALUE and *"EXTERNAL"* RATE OF RETURN AN ALTERNATIVE PEDOGY FOR TEACHING DISCOUNTED CASH FLOW ANALYSIS

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ABSTRACT

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AN ALTERNATIVE PEDOGY FOR TEACHING DISCOUNTED CASH FLOW ANALYSIS

The traditional content of and pedagogy for teaching Discounted Cash Flow (DCF) Analysis includes the investment performance measures of Net Present Value (NPV), Profitability Index (PI), and Internal Rate of Return (IRR), at a minimum, and may also include Adjusted or Modified Internal Rate of Return (AIRR or MIRR). This paper offers an alternative definition of and pedagogy for teaching these financial concepts. For example, it is demonstrated herein that:

(1) Net Present ***Value*** (NPV) ***is not*** a value;
(2) ***Profit***ability Index (PI) does ***not*** measure profit;
(3) ***Internal*** Rate of Return (IRR) ***usually is not*** a return on the Initial Investment;
(4) ***Gross*** Present Value (GPV) ***is a prerequisite*** to ***Net*** Present Value and it, unlike NPV, ***is*** a value; and
(5) A full and complete understanding of the ***Internal*** Rate of Return (IRR) requires an understanding of the ***External*** Rate of Return (ERR)!

This paper also offers a set of concatenation statements which link the so noted financial measures of investment performance one to another, thereby making the pedagogy more robust!

INTRODUCTION

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TRADITIONAL PEDAGOGY

The typical finance course/professor presentation states that the NPV is the ***difference*** between the present value of future cash flows one (CF_1) through the last (CF_n) all discounted at the investors hurdle or discount rate (i), and the initial investment symbolized by cash flow zero (CF_0). The NPV equation, therefore, may be written as follows:

$$NPV = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_n}{(1+i)^n} - CF_0$$

Known: $CF_{1\text{-}n}$; i; CF_0 (right-hand side of equation)
Unknown: NPV (left-hand side of equation)

The PI equation uses the same numbers as the NPV equation uses, however, CF_0 is not subtracted from the present value of the cash flows, but rather is divided into them, forming a ratio as follows:

$$PI = \frac{\frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_n}{(1+i)^n}}{CF_0}$$

Known: $CF_{1\text{-}n}$; i; CF_0 (right-hand side of equation)
Unknown: PI (left-hand side of equation)

The IRR equation uses the same cash flows ($CF_{1\text{-}n}$) as the NPV and PI equations. However, instead of the discount being a known input provided by the investor, the i is the unknown discount rate that balances the IRR equation, i.e., makes the right-hand side of the IRR equation equal the left-hand side of the IRR equation as follows:

$$CF_0 = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_n}{(1+i)^n}$$

Known: CF_0; (left-hand side of equation)
CF_{1-n} (right-hand side of equation)
Unknown: IRR = i (right-hand side of equation)

Some IRR presentations write the IRR equation as an NPV equation equal to zero (0), stating that the IRR equals the discount rate (i) that makes the NPV equal zero (0), as follows:

$$0 = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_n}{(1+i)^n} - CF_0$$

Known: CF_{1-n}; CF_0 (right-hand side of equation)
NPV = 0 (left-hand side of equation)
Unknown: IRR = i (right-hand side of the equation)

This equation defines the IRR in terms of the NPV. Likewise, the following equation writes the IRR equation as a PI equation equal to one (1.0), stating that the IRR equals the discount rate (i) that makes the PI equal one (1.0), as follows:

$$1.0 = \frac{\frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_n}{(1+i)^n}}{CF_0}$$

Known: CF_{1-n}; i; CF_0 (right-hand side of equation)
PI = 1.0 (left-hand side of equation)
Unknown: IRR = i (right-hand side of equation)

In the opinion of many, to define one complex term (IRR) in terms of another complex term (NPV or PI) only further complicates the teaching and understanding of former term (IRR). Better to define any and all terms in the simplest manner. To do otherwise, would be a blatant violation of the so-called Occam's Razor.[1] Occam's Razor, also known as the Principle of Parsimony, is the meta-theoretical principle that may be translated as:

(1) "entities must not be multiplied beyond necessity"; or

(2) "a plurality (of reasons) should not be posited without necessity." or more simply;

(3) "Keep It Simple, Stupid!" The so-called "KISS Principle".

Occam suggested that the simplest solution is usually the best and/or the most correct. The alternative pedagogy presented herein for discounted cash flow analysis is meant to follow the tenets of Occam; there will be no violations of his Razor in this paper!

TRADITIONAL (MIS)UNERSTANDING OF DCF ANALYSIS & ALTERNATIVE PEDAGOGY

The DCF equations presented supra usually lead the student (and often the professor, too) to draw the following conclusions about the understanding of the DCF concepts and equations:

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[1] William of Occam, England, b. c.1285, d. c.1349, ranks among the most important philosopher-theologians and one of the greatest logicians of the Middle Ages.

(1) Most texts and professors never discuss Gross Present Value. Yet, GPV is a prerequisite to NPV, as demonstrated infra.

(2) NPV is a value. Value is the noun modified by two adjectives: Net and Present. However, NPV is ***not*** a value, thereby making the adjectives and the term NPV irrelevant and misunderstood, as demonstrated infra.

(3) PI measures profit. Profit is defined as Revenues minus Expenses. PI is NOT a measure of profit, as demonstrated infra.

(4) IRR is a return on Cash Flow Zero (CF_0), the Initial Investment. This is seldom the case, as demonstrated infra.

(5) IRR assumes reinvestment at the IRR, while NPV assumes reinvestment at the Discount Rate. In fact, neither IRR nor NPV assume anything about reinvestment. The proof of the lack of reinvestment is demonstrated infra.

Exhibit I is offered as an ***alternative pedagogy*** for teaching Discounted Cash Flow (DCF) Analysis. The equations for each of the DCF metrics are linked to one another via sets of numerical and verbal concatenation statements. **Exhibit II** will present more information regarding the IRR and the matter of reinvestment.

EXHIBIT I

ACRONYMS FOR INVESTMENT DECISION MAKING DISCOUNTED CASH FLOW (DCF) ANALYSIS

1. WHAT IS ITS WORTH, VALUE, OR GROSS PRESENT VALUE? GPV

2. SHOULD I BUY IT? V-PD; V/PR

3. IF I BUY IT, WHAT RATE OF RETURN WILL I EARN? IRR

The DCF equations for the Gross Present Value (GPV), the Value-Price Differential (V-PD), and the Value/Price Ratio (V/PR), where the discount rate (i) is the ***known or chosen*** discount rate, and the DCF equation for the Internal Rate of Return (IRR), where the discount rate (i) is the ***unknown or calculated*** rate that balances the IRR equation answer these questions and are displayed infra.

1. HOW MUCH IS IT WORTH? WHAT'S ITS VALUE? GPV

GROSS PRESENT VALUE (DCF VALUE) EQUATION

$$GPV = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \dots + \frac{CF_n}{(1+i)^n}$$

Known: CF_{1-n} ; i
Unknown: PV

*GPV = DCF Value; Value

Note: Price (CF_0) is *not* part of the equation!

2. SHOULD I BUY IT? V-PD; V/PR

VALUE-PRICE DIFFERENTIAL (V-PD)* EQUATION
(Comparing Value and Price via Subtraction)

$$V\text{-}PD = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \dots + \frac{CF_n}{(1+i)^n} - CF_0$$

Known: CF_{0-n}; i
Unknown: V-PD

* V-PD = GPV – CF_0
V-PD = Value – Price

VALUE/PRICE RATIO (V/PR)* EQUATION
(Comparing Value and Price via Division)

V/PR = GPV / CF_0

Known: CF_{0-n}; i
Unknown: V/PR

* V/PR = Value / Price

3. IF I BUY IT, WHAT RATE OF RETURN WILL I EARN? IRR

INTERNAL RATE OF RETURN (IRR = i)* EQUATION

$$CF_0 = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \dots + \frac{CF_n}{(1+i)^n}$$

Known: CF_{0-n}
Unknown: IRR = i

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*@ i, CF_0 = GPV; or Price = Value; or Price = DCF Value
The IRR is the *calculated (not chosen)* discount rate (i) that makes Value *equal* Price, ie., the *calculated* discount rate that makes the equation's RHS = the equation's LHS.

SOME OBSERVATIONS

1. GPV is the value of any and all investments. GPV has an equation that is different than the NPV equation. Most texts and professors do not discuss/teach GPV, per se. The calculation of value (GPV) is rudimentary to all investment decisions. One cannot "buy low and sell high" without value. Price, symbolized as CF_0, is ***NOT*** a part of the GPV equation (Price is not an input to get Value). However, a calculator or a spread sheet might require CF_0 to be an input of zero, or minus zero, simply to get to CF_1. However, GPV is not baseball, wherein one must go to 1st base (CF_0), in order to get to 2nd base (CF_1). Some people think CF_0 is part of the Value (GPV) equation, due to it having to be inputted to get to CF_1. By looking at the GPV equation supra, one can see that Price (CF_0) is **not** in the GPV equation, even if a calculator requires it to be inputted, and therefore, is not an input to get Value!
2. Net Present Value (NPV) is ***NOT*** a Value. Rather, the NPV simply is the difference between Value and Price (NPV = GPV – CF_0). Hence, NPV may be better described as the **Value-Price Differential (V-PD)!** A differential is not a value of anything. For example, the difference between one's weight and another's weight is **not** the weight of either. Since a chosen discount rate is employed to calculate GPV and NPV, and since the IRR equation **finds** the discount rate that makes the right-hand side of the IRR equation (an iterated GPV) equal the left-hand side of the IRR equation (Price, Cost, Initial Investment, CF_0) , NPV simply is an equation that reveals the relationship between the **chosen** Discount Rate (used to calculate the GPV) and the **calculated** (via iteration) IRR. Hence, the NPV is a relative measure of two rates of return, namely the Chosen Discount Rate and the Calculated IRR. Who knew that not only is the NPV **not** a value, but also, the NPV is a measure (a relative measure) of rate of return!?! NPV is a metric that simply indicates that the investor is earning an IRR that is either above (a positive NPV), below (a negative NPV), or equal to (a zero NPV) the investor's chosen discount or hurdle rate! Therefore, Value-Price Differential (V-PD) more correctly describes what the world of finance and investment has long-called the NPV. If the V-PD is a positive differential, then the Calculated IRR is greater than the Discount Rate chosen to calculate the GPV. If the V-PD is a negative differential, then the Calculated IRR is less than the Discount Rate chosen to calculate the GPV. If the V-PD is a zero differential, then the Calculated IRR is exactly equal to the Discount Rate chosen to calculate the GPV.
3. The so-called Profitability Index (PI) does ***not*** measure profit! Rather, the PI simply is a ratio of GPV divided by Price (CF_0). Hence, PI may be better described as the **Value/Price Ratio (V/PR)!!** Said ratio does ***not*** measure profit. Profit is Revenues minus Expenses, not a ratio of Value (GPV) divided by Price (CF_0). Since a **chosen** discount rate is employed to calculate the

GPV numerator of the V/PR, and the IRR is the **calculated** discount rate that balances the IRR equation, then once the IRR is found, the right-hand side of the IRR equation is a GPV at the IRR. Hence, like the NPV, the PI is a relative measure of two rates of return, namely the Chosen Discount Rate and the Calculated IRR. Who knew that not only is the PI **not** a measure of profit, but also the PI (like the NPV) is a measure (a relative measure) of rate of return!?! If the V-PR is greater than one (1.0), then the Calculated IRR is greater than the Discount Rate chosen to calculate the GPV numerator of the V-PR. If the V-PR is less than one (1.0), then the Calculated IRR is less than the Discount Rate chosen to calculate the GPV. If the V-PR is exactly equal to one (1.0), then the Calculated IRR is exactly equal to the Discount Rate chosen to calculate the GPV. Finally, none of what has been said here about the NPV and the PI is reason to define the IRR in terms of either the NPV or the PI. The terms V-PD and V/PR are ***not*** (and should ***not*** be) defined in terms of the IRR.

4. Most texts and professors state that the calculation of the IRR includes (assumes) reinvestment of the cash flows ($CF_{1\text{-}n}$) at the IRR, and that the calculation of the NPV includes (assumes) reinvestment of the cash flows ($CF_{1\text{-}n}$) at the Discount Rate. Nothing could be farther from the truth! The issue of reinvestment being generally misunderstood and incorrectly taught around the world has been addressed most recently in the Journal of Real Estate Portfolio Management and the Indian Journal of Economics and Business by yours truly. A brief summary of these articles, including Exhibits II and III, follows herein. In a nutshell, said articles present "profiling the IRR and defining the ERR" (External Rate of Return) to demonstrate that the IRR:
   a. seldom is a return "on" the Initial Investment (Price, Cost, CF_0); and
   b. does ***not include*** reinvestment, and
   c. the ERR ***does include*** reinvestment.

PROFILING THE IRR

Profiling the IRR involves separating each cash flow into return "on" investment and return "of" investment in the same manner as a loan amortization schedule separates the interest and principal portions of each loan payment. **Exhibit II** displays the profiling of five (A – E) investments. With an amortized loan, the return "on" investment (interest) is earned on the changing outstanding loan balance, which in **Exhibit II** is called the "Outstanding Internal Investment". An amortized loan balance changes negatively or positively depending on whether there is positive or negative loan amortization. Said amortization is a function of the magnitude of each cash flow relative to the product of the interest rate times the loan balance. Amortization in **Exhibit II** and **Exhibit III** is called capital recovery which is also positive or negative, depending on the magnitude of each cash flow relative to the Outstanding Internal Investment. The profiling process, therefore, has five steps as follows:

1. Calculate the IRR;
2. Multiply the IRR x Outstanding Internal Investment = Return “On”;
3. Subtract Return “On” from Cash Flow = Return “Of”;
4. Subtract Return “Of” from Outstanding Internal Investment = Profile Number;
5. Repeat Steps 2-4 = IRR Profile Numbers “on” which the IRR is a return for each time period.

EXHIBIT II - PROFILING THE IRR - "IT'S ALL INSIDE"*

THE IRR USUALLY IS NOT A RETURN ON CF_0

Investment A

$$100{,}000 = \frac{43{,}798}{(1+i)^1} + \frac{43{,}798}{(1+i)^2} + \frac{43{,}798}{(1+i)^3}$$

IRR = 15%

IRR PROFILE	OUTSTANDING "INTERNAL" INVESTMENT	15% IRR RETURN "ON"		RETURN "OF"		SUM OF CF_{1-n}	
Decreasing	100,000 x .15 =	15,000	+	28,798	=	**43,798**	**CF_1**
Internal	71,202 x .15 =	10,681	+	33,117	=	**43,798**	**CF_2**
Investment	38,085 x .15 =	5,713	+	38,085	=	**+43,798**	**+ CF_3**
	0	31,394	+	100,000	=	**131,394**	**"on"+"of"**

Investment B

$$100{,}000 = \frac{5{,}000}{(1+i)^1} + \frac{5{,}000}{(1+i)^2} + \frac{139{,}725}{(1+i)^3}$$

IRR = 15%

IRR PROFILE	OUTSTANDING "INTERNAL" INVESTMENT	15% IRR RETURN "ON"		RETURN "OF"		SUM OF CF_{1-n}	
Increasing	100,000 x .15 =	15,000	+	(10,000)	=	**5,000**	**CF_1**
Internal	110,000 x .15 =	16,500	+	(11,500)	=	**5,000**	**CF_2**
Investment	121,500 x .15 =	18,225	+	121,500	=	**139,725**	**+ CF_3**
	0	49,725	+	100,000	=	**149,725**	**"on"+"of"**

Investment C

$$100{,}000 = \frac{20{,}000}{(1+i)^1} + \frac{10{,}000}{(1+i)^2} + \frac{114{,}138}{(1+i)^3}$$

IRR = 15%

IRR PROFILE	OUTSTANDING "INTERNAL" INVESTMENT	15% IRR RETURN "ON"		RETURN "OF"		SUM OF CF_{1-n}	
Decreasing	100,000 x .15 =	15,000	+	5,000	=	**20,000**	**CF_1**
-Increasing	95,000 x .15 =	14,250	+	(4,250)	=	**10,000**	**CF_2**
Internal	99,250 x .15 =	14,888	+	99,250	=	**114,138**	**+ CF_3**
Investment	0	44,138	+	100,000	=	**144,138**	**"on"+"of"**

Investment D

$$100{,}000 = \frac{10{,}000}{(1+i)^1} + \frac{20{,}000}{(1+i)^2} + \frac{115{,}863}{(1+i)^3}$$

IRR = 15%

IRR PROFILE	OUTSTANDING "INTERNAL" INVESTMENT	15% IRR RETURN "ON"		RETURN "OF"		SUM OF CF_{1-n}	
Increasing-	100,000 x .15 =	15,000	+	(5,000)	=	**10,000**	**CF_1**
Decreasing	105,000 x .15 =	15,750	+	4,250	=	**20,000**	**CF_2**
Internal	100,750 x.15 =	15,113	+	100,750	=	**115,863**	**+ CF_3**
Investment	0	45,863	+	100,000	=	**145,863**	**"on"+"of"**

Investment E

$$100{,}000 = \frac{15{,}000}{(1+i)^1} + \frac{15{,}000}{(1+i)^2} + \frac{115{,}000}{(1+i)^3}$$

IRR = 15%

IRR PROFILE	OUTSTANDING "INTERNAL" INVESTMENT	15% IRR RETURN "ON"		RETURN "OF"		SUM OF CF_{1-n}	
Uncommon	100,000 x .15 =	15,000	+	0	=	**15,000**	**CF_1**
Case: CF_0	100,000 x .15 =	15,000	+	0	=	**15,000**	**CF_2**
Is Internal	100,000 x .15 =	15,000	+	100,000	=	**115,000**	**+ CF_3**
Investment	0	45,000	+	100,000	=	**145,000**	**"on"+"of"**

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* J. C. Penny advertising tag line.

In **Exhibit II**, “on” what the IRR is a return varies among the five investments as follows:

a. Investment A’s IRR is a return on a ***declining*** Outstanding Internal Investment;
b. Investment B’s IRR is a return on an ***increasing*** Outstanding Internal Investment;
c. Investment C’s IRR is a return on a ***decreasing-increasing*** Outstanding Internal Investment;
d. Investment D’s IRR is a return on an ***increasing-decreasing*** Outstanding Internal Investment; and
e. Investment E’s IRR is a return on CF_0, however, this is rarely the case due to the unlikely assumption that each an investment’s cash flows (except the last) will be equal to the product of the IRR and CF_0 and the last cash flow will equal the sum of CF_0 and the product of the IRR and CF_0, such that there will be no capital recovery until CF_0 has been received!

EMPLOYING THE EXTERNAL RATE OF RETURN (ERR)

Given complete and proper understanding of both the IRR and the ERR, one might choose to employ the ERR in the investment decision-making process. In order to make a fair comparison between two mutually-exclusive investment choices, wherein the magnitude and timing of the cash flows are different, one might choose to make an explicit assumption regarding the reinvestment of intermediate cash flows and the rate at which said cash flows might be reinvested up to the terminal date of the longer-lived investment. A fair comparison of Investments A and B from Exhibit II will demonstrate this point. The impact of reinvestment between these two investments is the most dramatic of any pair of investments in Exhibit II. Investments A and B have been selected to demonstrate the importance of reinvestment because their cash flows represent the most extreme differences, vis a’ vis magnitude and timing, among the five sets of cash flows. Their respective cash flows are as follows:

Investment A $$100{,}000 = \frac{43{,}798}{(1+i)^1} + \frac{43{,}798}{(1+i)^2} + \frac{43{,}798}{(1+i)^3}$$ **IRR = 15%**

Investment B $$100{,}000 = \frac{5{,}000}{(1+i)^1} + \frac{5{,}000}{(1+i)^2} + \frac{139{,}725}{(1+i)^3}$$ **IRR = 15%**

Investments A and B from Exhibit II both have IRRs of 15%. To demonstrate that **reinvestment does matter**, the cash flows of these two investments have been reinvested at three different reinvestment rates (5%; 15%; and 25%) to obtain three different terminal values, which are then used with the initial investment of 100,000 to calculate the three respective ERRs. These reinvestment and ERR calculations are presented in **Exhibit III**. Note that reinvesting at the 15% IRR results in an ERR equal to the IRR. However, this phenomenon does **not** prove (as some people think) that one **must** reinvest at the IRR to calculate the IRR. Rather, it simply demonstrates that a weighted average remains the same if more of the same numbers are added and averaged. For example, if a baseball player’s batting average is .333 and

EXHIBIT III

REINVESTMENT AND THE EXTERNAL RATE OF RETURN (ERR)

INVESTMENT A

Reinvest at Three Different Rates

Compound→ leaving 0's	at 5%	at 15%	at 25%
→→→→→→→→→→→→→→→→→→→→→	48,287	57,923	68,434
↑ →→→→→→→→→→→→→→	45,988	50,367	54,748
↑ ↑ →→→→→→→	+ 43,798	+ 43,798	+ 43,798
↑ ↑ ↑	138,073	152,088	166,980

$$100{,}000 = \frac{43{,}798}{(1+i)^1} + \frac{43{,}798}{(1+i)^2} + \frac{43{,}798}{(1+i)^3}$$
No Reinvestment
IRR = 15.00%

$$100{,}000 = \frac{0}{(1+i)^1} + \frac{0}{(1+i)^2} + \frac{138{,}073}{(1+i)^3}$$
Reinvest at 5%
ERR = 11.35%

$$100{,}000 = \frac{0}{(1+i)^1} + \frac{0}{(1+i)^2} + \frac{152{,}088}{(1+i)^3}$$
Reinvest at 15%
ERR = 15.00%

$$100{,}000 = \frac{0}{(1+i)^1} + \frac{0}{(1+i)^2} + \frac{166{,}980}{(1+i)^3}$$
Reinvest at 25%
ERR = 18.64%

INVESTMENT B

Reinvest at Three Different Rates

Compound→ leaving 0's	at 5%	at 15%	at 25%
→→→→→→→→→→→→→→→→→→→→	5,513	6,613	7,813
↑ →→→→→→→→→→→→→→	5,250	5,750	6,250
↑ ↑ →→→→→→→	+139,725	+139,725	+139,725
↑ ↑ ↑	150,488	152,088	153,788

$$100{,}000 = \frac{5{,}000}{(1+i)^1} + \frac{5{,}000}{(1+i)^2} + \frac{139{,}725}{(1+i)^3}$$
No Reinvestment
IRR = 15%

$$100{,}000 = \frac{0}{(1+i)^1} + \frac{0}{(1+i)^2} + \frac{150{,}488}{(1+i)^3}$$
Reinvest at 5%
ERR = 14.60%

$$100{,}000 = \frac{0}{(1+i)^1} + \frac{0}{(1+i)^2} + \frac{152{,}088}{(1+i)^3}$$
Reinvest at 15%
ERR = 15.00%

$$100{,}000 = \frac{0}{(1+i)^1} + \frac{0}{(1+i)^2} + \frac{153{,}788}{(1+i)^3}$$
Reinvest at 25%
ERR = 15.43%

s/he plays (reinvests in) additional games and bats 1 for 3 in each additional game that is external to the original number of games, the total overall batting average will still be .333! However, said total or overall batting average is based on the combined result of all of the games played and, therefore, more times at bat. In other words, the ERR of any given investment is the combined result, or weighted average, of the IRRs on more than just the original investment being analyzed, namely those additional ones made by reinvesting the original investment's cash flows. Hence, **Exhibit III** clearly demonstrates that **reinvestment does matter** in the **application** of the IRR, but **absolutely has nothing to do with the calculation of the IRR**. In other words, **reinvestment is integral to <u>only</u> the ERR!!**

It is also worth noting that the differences in the magnitude and timing of the cash flows in Investments A and B cause the two ranges of ERRs to be quite different from one another. For Investment A, the ERR range is 7.29% (11.35% - 18.64%); while for Investment B, the ERR range is only 0.83% (14.60% - 15.43%). The meaning, significance, and application of these numbers are the subject of another paper. However, suffice it to say that the two IRRs of 15% in Investments A and B are not as similar, let alone identical, as they might appear to be prior to the calculation of their respective ERRs.

Finally, any financial analyst must fully and completely understand **both** what the IRR **is** and what it **is not,** in order to properly interpret its meaning and to skillfully apply it in practice. The IRR is not exactly what many have come to think it is, and it **assumes nothing regarding reinvestment.** Only the analyst can make the reinvestment assumption, if any, for the application in a portfolio context. S/he must compare the **IRR Profiles** of seemingly similar IRRs before concluding that they are virtually identical; they may be quite different! As they say in Thailand: "Same, Same, But Different"!

<u>PROFILING SUMMARY AND CONCLUSION</u>

To summarize, Profiling the IRR provides that, if in any given time-period the cash flow received is more (less) than is needed to earn the IRR for that period, then the extra (deficient) amount of cash flow is attributed to positive (negative) capital recovery, recapture, or amortization. Said amount of capital recovery effectively decreases (increases) the amount of capital **remaining internal** to the investment. This process of capital recovery is analogous to positive (negative) amortization of a fully-amortized loan investment!

Profiling the IRR reveals the true meaning of the IRR as follows:

1. It is seldom a return "on" the initial investment (CF_0) in each period;
2. It is usually a return "on" varying amounts in each period;

3. There is **no reinvestment** in its **calculation**;
4. Reinvestment should be considered in the application of the IRR in order to make a fair comparison between two or more mutually-exclusive investments, especially if terminal-value wealth-maximization is a stated objective of the investor;
5. If and when reinvestment is employed, the resulting rate of return is no longer an internal rate of return (IRR), but rather an external rate of return (ERR) which is a function of the reinvestment rate **chosen** by the investor;
6. Profiling the IRR unequivocally settles the reinvestment rate controversy: the **internal rate of return (IRR) does not** assume, nor require, reinvestment of the intermediate cash flows; whereas the **external rate of return (ERR) does** assume, and require, reinvestment of the intermediate cash flows!

FINAL THOUGHTS FOR THE FUTURE

It is worth noting that over the past 40 years, I have had many financed-trained students who have stated that they wish they had learned DCF Analysis via the alternative pedagogy offered herein. One might be very surprised and quite puzzled that the misunderstandings and faulty pedagogies of DCF Analysis have lasted so long. It is hoped that this paper might be a step in the right direction. A key point to remember while reading this paper, is that the new terminology and terms herein actually mean what the words say and say what the words mean!