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Global Trade and Economic Growth with Endogenous Knowledge

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Abstract

The purpose of this paper is to study global growth with capital accumulation and knowledge creation. We show how the world income distribution and trade pattern evolve over time when capital accumulation and learning-by-doing are the engines of global economic growth with free trade. The countries differ in preference, knowledge utilization efficiency and creativity. First, we show that the dynamics of the J -country world economy is described by $(J+1)$-dimensional differential equations. Then, we simulate the motion of the global economy with three economies, respectively called developed, industrializing, and underdeveloped economies (DE, IE, UE). We carry out comparative dynamic analysis with regard to changes in knowledge utilization efficiency, propensities to save, and the populations. We show that, for instance, when the DE increases its population, all countries benefit; when the UE increases its population, all the economies suffer in terms of per capita wealth and consumption levels in the long term.

1 Introduction

Irrespective of the fact that it has experienced an unprecedented increase in the production capacity in the last hundred years, the world has also shown a large disparity over space in income per capita and living standard. The diverging evidences imply that it is possible that globalization has negative impact on some groups of households in some countries. As mentioned by Findlay[1], one topic that was almost entirely absent from the pure theory of international trade was any consideration of the connection between economic growth and international trade in the classical literature of economic theory. Almost all the trade models developed before the 1960s are static in the sense that the supplies of factors of production are given and do not vary over time; the classical Ricardian theory of comparative advantage and the Heckscher-Ohlin theory are static since labor and capital stocks (or land) are assumed to be given and constant over time. It has only been in the last three or four decades that trade theory has made some systematic treatment of capital accumulation or technological changes in the context of international economics. In a neoclassical growth model it is possible for a poor economy to catch up with rich ones because of decreasing marginal product of capital. Rapid catch-up has often observed in the global economy in modern times. For instance, Table 1 gives the growth rates of European countries from 1875 to 1980. The economies are divided into three groups, basing on their income levels in 1875. By 1980 the income differences among the three groups are much smaller than in 1875. The per-capita income in the middle-income group was 60% of that in the UK in 1875 and 110% in 1980. The income level of the low-income group was 53% of the UK's, but became 90% in 1980[2]. Both the middle- and low-income groups had increased the per-capita incomes faster than the UK, even though the transition was not monotonic as shown in the table.

Table 1 Average Growth Rates (%) of European Economies (1875-1980)

Groups	**1875–1900**	**1900–25**	**1925–50**	**1950–60**	**1960–70**	**1970–80**
UK	1.17	0.85	0.85	2.37	2.26	1.76
Middle-income group	1.36	1.38	0.82	4.60	4.06	2.55
Low-income group	1.07	0.89	0.80	3.45	5.30	2.76

Source: [2]
Note: The middle-income group includes France Germany, low countries and Scandinavian countries, and the low-income group the rest of Western Europe.

It is important to examine the possible effects of trade upon national and personal income distribution in a globalizing world economy. The impact of trade on income inequality has been widely discussed in both academic and policy forums. The topic has increasing caused the attention partly because there is a great concern among rich economies about their ability to sustain living standards as some developing economies are experiencing fast economic growth. It has been argued that productivity differences explain much of the variation in incomes across countries, and technology plays a key role in determining productivity.[2, 3, 4, 5] The pattern of worldwide technical change is determined largely by international technology diffusion because a few rich countries account for most of the world's creation of new technology. Basing on the dataset on imports of technology and total factor productivity (TFP) over 135 years for the OECD countries, Madsen[6] empirically shows that knowledge spillovers have been a significant contributor to the TFP convergence among the OECD countries over the period of 1870 to 2004. Using a multi-sector version of the Ricardo-Viner model of international trade to empirically identify the effects of technological change and international trade on the US wage premium, Blum[7] finds that capital was reallocated to sectors where it is relatively complementary to skilled workers. As globalization is deepening, it is important to provide analytical frameworks for analyzing global economic interactions not only among developed economies, but treating the world as an integrated whole. For instance, it is important to examine how a developing economy like India or China may affect different economies as its technology is improved or population is increased; or how trade patterns may be affected as technologies are further improved or propensities to save are reduced in developed economies like the US or Japan.

Most of trade models with endogenous capital and/or knowledge are either limited to two-country or small open economies.[8, 9, 10] This study proposes a multi-country trade model with capital accumulation and knowledge creation in a perfectly competitive global economy. The model is based on neoclassical growth theory with capital accumulation and growth theory with endogenous knowledge. Trade models with capital movements are originated by MacDougall[11] and Kemp[12], even though these models were limited to static and one-commodity frameworks. A dynamic model, which takes account of accumulating capital stocks and of growing population within the Heckscher-Ohlin type of model, was initially developed by Oniki and Uzawa and others[13, 14], in terms of the two-country, two-good, two-factor model of trade. The model is primarily concerned with the process of world capital accumulation and distribution with demands and supplies as fast processes. The two-sector growth model has often been applied to analyze the interdependence between trade patterns and economic growth. These models are used to study the dynamics of capital accumulation and the various balance of payments accounts. There are different sets of assumptions made about the structure of trade. For instance, in the trade models by, free trade in both consumption and investment goods is allowed. An alternative specification of trade structure in the growth framework allows for the existence of international financial markets and for free trade in consumption goods and securities, but not in investment goods.[8, 15, 16, 17]. It should be noted that a trade model based on the Solow approach is proposed by Sorger[18]. Vellutini[19] proposes a trade model to examine possible poverty traps due to capital mobility. This framework emphasizes the interaction of foreign borrowing, debt service, and domestic capital accumulation. The two-sector neoclassical growth theory was also applied to analyzing small open economies. Irrespective of analytical difficulties involved in analyzing two-country, dynamic-optimization models with capital accumulation, many efforts have been made to examine the impact of saving, technology, and various policies upon trade patterns within this framework.[20, 21, 22, 23] As far as capital accumulation and trade pattern determination are concerned, our study follows the Oniki-Uzawa framework[24], even though this study deviates from the traditional approach in modeling behavior of households. We analyze trade issues within the framework of a simple international macroeconomic growth model with perfect capital mobility.

Trade economists have recently developed different trade models in which endogenous growth is generated either by the development of new varieties of intermediate or final goods or by the improvement of an existing set of goods with endogenous technologies[25-19]. These studies attempted to formalize trade patterns with endogenous technological change and monopolistic competition. They often link trade theory with increasing-returns growth theory. Within such frameworks the dynamic interdependence between trade patterns, R&D efforts, and various economic policies are connected. With the development of models with endogenous long-run growth, economists now have formal techniques with which to explore the relationship between trade policy and long-run growth either with knowledge or with capital, but in most of them not with both capital and knowledge within the same framework. It is well known that dynamic-optimization models with capital accumulation are associated with analytical difficulties. To avoid these difficulties, this study applies an alternative approach to consumer behavior. It will be demonstrated that the multi-country trade model with capital accumulation and knowledge creation becomes analytically tractable with the new approach to consumer behavior. The model in this study is a further development of the two model by Zhang[30]. This study models behavior of consumers different from the previous study. Moreover, the previous study was only concerned with examining equilibrium and comparative statics analysis. As no simulation was provided in the previous study, it is almost impossible to see how the multi-country system moves over time. This study simulates the model to see how the system moves over time and how the motion of the system is affected when some parameters are changed. This paper is organized as follows. Section 2 defines the multi-country model with capital accumulation and knowledge creation. Section 3 shows that the dynamics of the world economy with J countries can be described by $(J+1)$-dimensional differential equations. As mathematical analysis of the system is too complicated, we demonstrate some of the dynamic properties by simulation when the world economy consists of three countries. Sections 4 – 6 examine respectively effects of changes in knowledge utilization efficiency, the population, and the preference. Section 7 concludes the study. The analytical results in Section 3 are proved in Appendix A1. Appendix A2 examines the case when all the countries have the same preference.

2 The multi-country trade model with capital and knowledge

The study used the traditional approach to household behavior as in the Solow one-sector growth model, assuming a constant fraction using for saving. The knowledge creation is only through Arrow's learning by doing. Each country has one production sector. Knowledge growth is through learning by doing. We consider knowledge as an international public good in the sense that all countries access knowledge and the utilization of knowledge by one country does not affect that by others. In describing the production sector, we follow the neoclassical trade framework. It is assumed that the countries produce a homogenous commodity. Most aspects of production sectors in our model are similar to the neo-classical one-sector growth model[31]. There is only one (durable) good in the global economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. All savings volunteered by households are absorbed by firms. The system consists of multiple countries, indexed by $j = 1, ..., J$. Each country has a fixed labor force, N_j, ($j = 1, ..., J$). Let $K_j(t)$ and $F_j(t)$ stand for respectively the capital stocks employed by and the output level of the production sector by country j.

Let prices be measured in terms of the commodity and the price of the commodity be unity. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j th country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$.

Behavior of producers

First, we describe behavior of the production sections. We assume that there are three factors, physical capital, labor, and knowledge at each point of time t. The production functions are given by

$$F_j(t) = A_j Z^{m_j}(t) K_j^{\alpha_j}(t) N_j^{\beta_j}(t), \ \ A_j > 0, \ \ \alpha_j + \beta_j = 1, \ \ \alpha_j, \beta_j > 0, \ \ j = 1, \cdots, J,$$

in which $Z(t)\ (> 0)$ is the world knowledge stock at time t. Here, we call m_j country j's knowledge utilization efficiency parameter. If we interpret $Z^{m_j/\beta}N_j$ as country j's human capital or qualified labor force, we see that the production function is a neoclassical one and homogeneous of degree one with the inputs. As cultures, political systems and educational and training systems vary between countries, m_j are different.

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest and wage rates are determined by markets. Hence, for any individual firm $r(t)$ and $w_j(t)$ are given at each point of time. The production sector chooses the two variables $K_j(t)$ and $N_j(t)$ to maximize its profit. The marginal conditions are given by

$$r + \delta_{kj} = A_j \alpha_j Z^{m_j} k_j^{-\beta_j}, \ \ w_j = A_j \beta_j Z^{m_j} k_j^{\alpha_j}, \tag{1}$$

where δ_{kj} are the depreciation rate of physical capital in country j and $k_j(t) \equiv K_j(t)/N_j(t)$.

Behavior of consumers

Each worker may get income from wealth ownership and wages. Consumers make decisions on consumption levels of goods as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang in the early 1990s. Let $\bar{k}_j(t)$ stand for the per capita wealth in country j. Each consumer of country j obtains income

$$y_j(t) = r(t)\bar{k}_j(t) + w_j(t), \ \ j = 1, \cdots, J \tag{2}$$

from the interest payment $r\bar{k}_j$ and the wage payment w_j. We call y_j the current income in the sense that it comes from consumers' payment and consumers' current earnings from ownership of wealth. The sum of income that consumers are using for consuming, saving, or transferring are not necessarily equal to the temporary income because consumers can sell wealth to pay, for instance, the current consumption if the temporary income is not sufficient for buying food and touring the country. Retired people may live not only on the interest payment but also have to spend some of their wealth. The total value of wealth that a consumer of group j can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is equal to

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{3}$$

The disposable income is used for saving and consumption. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$), in the above equation is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute between consuming and saving. It should also be remarked that in the growth literature, for instance, in the Solow model, the saving is out of the current income, $y_j(t)$, while in this study the saving is out of the disposable income. This approach is discussed at length by Zhang[32]. Zhang has also examined the relations between his approach and the Solow growth theory, the Ramsey growth theory, the permanent income hypothesis, and the Keynesian consumption function in details.

At each point of time, a consumer distributes the total available budget between savings, $s_j(t)$, and consumption of goods, $c_j(t)$. The budget constraint is given by

$$c_j(t) + s_j(t) = \hat{y}_j(t) = r(t)\bar{k}_j(t) + w_j(t) + \bar{k}_j(t). \qquad (4)$$

At each point of time, consumers have two variables to decide. A consumer decides how much to consume and to save. Equation (4) means that consumption and savings exhaust the consumers' disposable personal income.

We assume that utility levels that the consumers obtain are dependent on the consumption level of commodity, $c_j(t)$, and the savings, $s_j(t)$. The utility level of the consumer in country j, $U_j(t)$, is specified as follows

$$U_j(t) = c_j^{\xi_{0j}}(t) s_j^{\lambda_{0j}}(t), \ \ \xi_{0j}, \lambda_{0j} > 0, \qquad (5)$$

where ξ_{0j} and λ_j are respectively household j 's propensities to consume and to hold wealth. Here, for simplicity, we specify the utility function with the Cobb-Douglas from. It would provide more insights if we take some other forms of utility functions. In this study we fix the preference structure. It is quite reasonable to assume that one's attitude towards the future is dependent on factors such as capital gains, the stock of durables, income distribution and demographic factors. Maximizing subject to the budget constraints (4) yields

$$c_j(t) = \xi_j \, \hat{y}_j(t), \ \ s_j(t) = \lambda_j \, \hat{y}_j(t), \qquad (6)$$

in which

$$\xi_j \equiv \frac{\xi_{0j}}{\xi_{0j} + \lambda_{0j}}, \ \ \lambda_j \equiv \frac{\lambda_{0j}}{\xi_{0j} + \lambda_{0j}}.$$

According to the definitions of $s_j(t)$, the wealth accumulation of the representative household in country j is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \qquad (7)$$

This equation simply states that the change in wealth is equal to the savings minus dissavings.

Knowledge creation and behavior of the university

Like capital, a refined classification of knowledge and technologies tend to lead new conceptions and modeling strategies. Some major new knowledge and inventions that had far reaching and prolonged implications, such as Newton's mechanics, Einstein's theory of relativity, steam engine, electricity, and computer. Small improvements and non-lasting improvements take place everywhere, serendipitously and intentionally. Innovations may also happen in a drastic, discontinuous fashion or in a slow, continuous manner. The introduction of the first steam engine rapidly triggered a sequence of innovations. The same is true about electricity and computer. Bresnahan and Trajtenberg[33] argued that technologies have a treelike structure, with a few prime movers located at the top and all other technologies radiating out from them. They characterize general purpose technologies by pervasiveness (which means that such a technology can be used in many downstream sectors), technological dynamism (which means that it can support continuous innovational efforts and learning), and innovational complementarities (which exist because productivity of R&D in downstream sectors increases as a consequence of innovation in the general purpose technology, and vice versa). This study uses knowledge in a highly aggregated sense. We assume that knowledge growth is through the learning by doing. We propose the following equation for knowledge growth

$$\dot{Z}(t) = \sum_{j=1}^{J} \frac{\tau_j F_j(t)}{Z^{\varepsilon_j}(t)} - \delta_z Z(t) \qquad (8)$$

in which δ_z (≥ 0) is the depreciation rate of knowledge, and ε_j, and τ_j are parameters. Equation (8) implies that knowledge accumulation is through learning by doing. The parameters τ_j and are non-negative. We interpret $\tau_j F / Z^{\varepsilon_j}$

as the contribution to knowledge accumulation through learning by doing by country $j's$ production sector. To see how learning by doing occurs, assume that knowledge is a function of country $j's$ total industrial output during some period

$$Z(t) = a_1 \left\{ \int_0^t F_j(\theta) d\theta \right\}^{a_2} + a_3$$

in which and are positive parameters. The above equation implies that the knowledge accumulation through learning by doing exhibits decreasing (increasing) returns to scale in the case of $a_2 < (>) \, 1$. We interpret a_1 and a_3 as the measurements of the efficiency of learning by doing by the production sector. Taking the derivatives of the equation yields $\dot{Z} = \tau_j F_j / Z^{\varepsilon_j}$, in which $\tau_j \equiv a_1 a_2$ and $\varepsilon_j \equiv 1 - a_2$.

The total capital stocks of the world, $K(t)$, is equal to the wealth owned by the world. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) \ = \sum_{j=1}^{J} \bar{k}_j(t) N_j . \tag{9}$$

The world production is equal to the world consumption and world net savings. That is

$$C(t) + S(t) - K(t) + \sum_{j=1}^{J} \delta_{kj} K_j(t) = F(t),$$

where

$$C(t) \equiv \sum_{j=1}^{J} c_j(t) N_j, \quad S(t) \equiv \sum_{j=1}^{J} s_j(t) N_j, \quad F(t) \equiv \sum_{j=1}^{J} F_j(t).$$

We have thus built the model with trade, economic growth, capital accumulation, knowledge creation and utilization in the world economy in which the domestic markets of each country are perfectly competitive, international product and capital markets are freely mobile and labor is internationally immobile.

4 The Global Economic Dynamics

Our model describes global economic dynamics for any number of countries. One may expect that it is difficult to study dynamic behavior of the model. We first show that in general case the dynamics of the world economy can be expressed by a $(J+1)-$ dimensional differential equations system.

Lemma 1
The dynamics of the world economy is governed by the following $(J+1)-$ dimensional differential equations system with $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$, $j = 2, \cdots, J$, as the variables

$$\dot{Z} = \Lambda(k_1, Z) = \sum_{j=1}^{J} \bar{\tau}_j Z^{m_j - \varepsilon_j} \phi_j^{\alpha_j} - \delta_z Z,$$

$$\dot{k}_1 = \Lambda_1\left(k_1, \{\bar{k}_j\}, Z\right) \equiv \left[\sum_{j=2}^{J} n_j \Lambda_j + \lambda_1 w_1 - n_0 R \psi - R \sum_{j=2}^{J} n_j \bar{k}_j - n_0 \psi_Z \Lambda \right] \frac{1}{n_0 \psi_{k_1}},$$

$$\dot{\bar{k}}_j = \Lambda_j\left(k_1, \bar{k}_j, Z\right) \equiv \lambda_j w_j - \left(1 - \lambda_j - \lambda_j r\right) \bar{k}_j, \quad j = 2, ..., J,$$

in which ϕ_j, R, Λ_j, ψ, ψ_Z, ψ_{k_1}, r and w_j are unique functions of $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$ at any point of time, defined in Appendix A1, and n_0, n_j, and $\bar{\tau}_j$ are parameters defined in Appendix A1. For any given positive values of $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$ at any point of time, the other variables are uniquely determined by the following procedure: $\bar{k}_1(t)$ by (A4) $\rightarrow$

$k_j(t)$, $j = 2, \cdots, J$ by (A1) $\to$ $r(t)$ by (1) $\to$ $w_j(t)$, $j = 1, \cdots, J$ by (A2) $\to$ $K_j(t) = k_j(t) N_j$ $\to$ $\hat{y}_j(t)$ by (A5) $\to$ $c_j(t)$ and $s_j(t)$ by (6) $\to$ $F_j = A_j Z^{m_j} K_j^{\alpha_j} N_j^{\beta_j}$.

We have the dynamic equations for the world economy with any number of countries. It should be noted that Appendix A1 examines the dynamic properties when the world population has an identical preference. It is demonstrated that the world economy is actually controlled only by two-dimensional differential equations. In this special case, we can analyse dynamic properties of the system. Nevertheless, when the countries have different preferences, the world economy cannot be described by two dimensional differential equations. The system is nonlinear and is of high dimension. It is difficult to generally analyze behavior of the system. We now solve equilibrium problem. For simplicity, we require $\delta_k = \delta_{kj}$, $j = 1, ..., J$. Equations (A1) and (A2) now become

$$k_j = \phi_j(k_1, Z) = \tau_{kj} Z^{\overline{m}_j} k_1^{\beta_1/\beta_j},$$
$$w_j = \overline{\phi}_j(k_1, Z) = \tau_{wj} Z^{m_{0j}} k_1^{\alpha_{wj}}, \quad j = 1, \cdots, J, \tag{10}$$

where

$$\tau_{kj} \equiv \left(\frac{A_j \alpha_j}{A_1 \alpha_1}\right)^{1/\beta_j}, \quad \overline{m}_j \equiv \frac{m_j - m_1}{\beta_j}, \quad \tau_{wj} \equiv A_j \beta_j, \quad m_{0j} \equiv m_j + \alpha_j \overline{m}_j, \quad \alpha_{wj} \equiv \frac{\beta_1 \alpha_j}{\beta_j}.$$

By equations (7), we have $s_j = \overline{k}_j$. By the definition of R and equations (1), we have

$$R(k_1, Z) = \lambda_1 \left(\lambda_{u1} - A_1 \alpha_1 Z^{m_1} k_1^{-\beta_1}\right), \tag{11}$$

in which $\lambda_{u1} \equiv 1/\lambda_1 - 1 + \delta_k$. From the equations for k_j in (10) and $K = \sum_{j=1}^{J} K_j$, we have

$$K = \psi = \sum_{j=1}^{J} \tau_{kj} N_j k_1^{\beta_1/\beta_j} Z^{\overline{m}_j}. \tag{12}$$

From $s_j = \overline{k}_j$ and equations (6), we have $\hat{y}_j = \overline{k}_j / \lambda_j$. Substitute $\hat{y}_j = \overline{k}_j / \lambda_j$ into (A7) at steady state

$$\overline{k}_j = \frac{\tau_{wj} Z^{m_{0j}} k_1^{\alpha_{wj}}}{\lambda_{uj} - \alpha_1 A_1 Z^{m_1} k_1^{-\beta_1}}, \quad j = 2, ..., J, \tag{13}$$

where we use (1) and $\lambda_{uj} \equiv 1/\lambda_j - 1 + \delta_k$. By equations (A12), at equilibrium we have

$$\Omega_k(k_1, Z) \equiv \lambda_1 w_1 - n_0 R \psi + R \sum_{j=2}^{J} n_j \overline{k}_j = 0, \tag{14}$$

in which we use $\Lambda = \Lambda_j = 0$. Substituting $k_j = \tau_{kj} Z^{\overline{m}_j} k_1^{\beta_1/\beta_j}$ into (A3) and setting the resulted equation at equilibrium, we have

$$\Omega_Z(k_1, Z) \equiv \sum_{j=1}^{J} \tau_j \tau_{kj}^{\alpha_{ij}} A_j N_j Z^{x_j} k_1^{\alpha_j \beta_1/\beta_j} - \delta_z = 0, \tag{15}$$

in which $x_j \equiv m_j - \varepsilon_j + \alpha_j \overline{m}_j - 1$. We see that two equations, $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, contain two variables, k_1 and Z. The two equations determine equilibrium values of k_1 and Z. By equations (13), we determine $\overline{k}_j$ for $j = 2, ..., J$. Following the procedure in Lemma 1, we determine all the other variables at equilibrium. We see that the main problem is to solve $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, for $k_1 > 0$ and $Z > 0$.

As we cannot explicitly solve the equilibrium values of k_1 and Z, we simulate the model to illustrate properties of the dynamic system. We specify the parameters as follows:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 8 \end{pmatrix}, \ \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.8 \\ 0.7 \end{pmatrix}, \ \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.1 \end{pmatrix}, \ \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.02 \end{pmatrix}, \ \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.32 \\ 0.31 \end{pmatrix},$$
$$\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.01 \\ 0.01 \end{pmatrix}, \ \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix}, \ \begin{pmatrix} \xi_{01} \\ \xi_{02} \\ \xi_{0.3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.25 \\ 0.3 \end{pmatrix}, \ \begin{pmatrix} \lambda_{01} \\ \lambda_{02} \\ \lambda_{03} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0.7 \\ 0.65 \end{pmatrix}, \ \delta_k = 0.05, \ \delta_Z = 0.04. \tag{16}$$

Country 1, 2 and $3's$ populations are respectively 3, 4 and 8. Country 3 has the largest population. Country 1, 2 and $3's$ total productivities, A_j, are respectively 1, 0.8 and 0.7. Country 1, 2 and $3's$ utilization efficiency parameters, m_j, are respectively 0.4, 0.2 and 0.1. Country 1 utilizes knowledge mostly effectively; country 2 next and country 3 utilizes knowledge lest effectively. We call the three countries respectively as developed, industrializing, and underdeveloped economies (DE, IE, UE). We specify the values of the parameters, α_j, in the Cobb-Douglas productions approximately equal to 0.3. [34] The DE's learning by doing parameter τ_{i1} is the highest among the countries. The returns to scale parameters in learning by doing, ε_{ij}, are all positive, which implies that knowledge exhibits decreasing returns to scale in learning by doing. The depreciation rates of physical capital and knowledge are specified respectively at 0.05 and 0.04. The DE's propensity to save is 0.75 and the UE's propensity to save is 0.65. The value of the IE's propensity is between the two other countries. We introduce country $j's$ returns to scale parameters for the production sector and the university respectively as follows:

$$x_j^* \equiv \frac{m_j}{\beta_j} - \varepsilon_j - 1, \quad j = 1, 2, 3.$$

which are respectively equal to, $-0.53, -0.91, -1.16$, with the specified values of the parameters. We have, $x_{ij}^* < 0$ for all j. As no economy in the global economy exhibits increasing returns to scale, it is expected that the dynamic system has a unique equilibrium point and it is stable. We now show that the dynamic system has a unique equilibrium point. Figure 1 plots the two equations, $\Omega_k(k_1, Z) = 0$ and $\Omega_Z(k_1, Z) = 0$, for $k_1 > 0$ and $Z > 0$. The solid lines represent $\Omega_k(k_1, Z) = 0$ and the dashed line stands for $\Omega_Z(k_1, Z) = 0$.

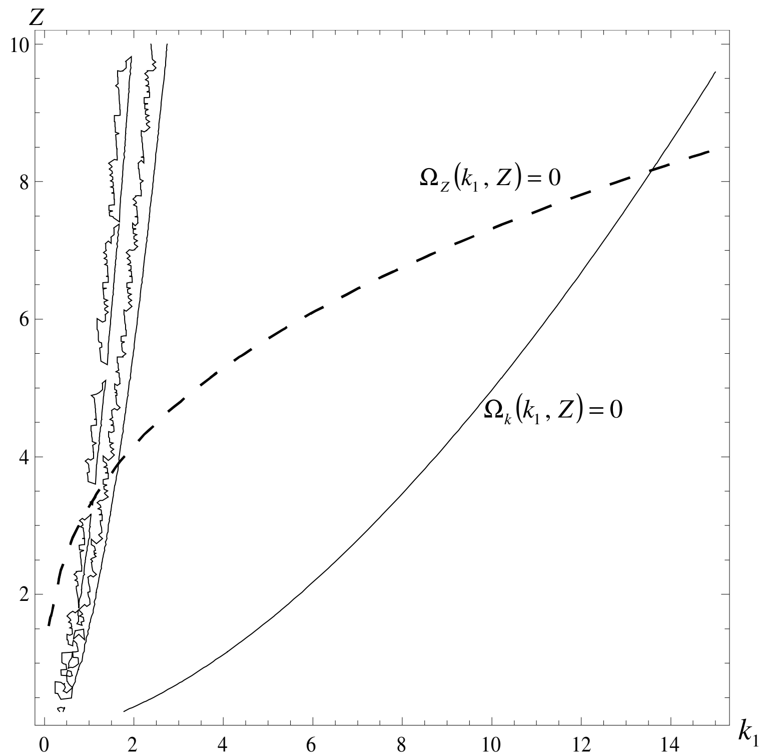

Figure 1 Solutions of Equations (14) and (15)

From Figure 1, we see that the two equations have multiple solutions. Nevertheless, it can be shown that only the following solution

$$k_1 = 13.58, \quad Z = 8.17.$$

is meaningful in the sense that all the other variables are economically meaningful. For instance, we also have a solution as $k_1 = 1.64$ and $Z = 3.89$. Nevertheless, this point is economically meaningless because at this point, we have $\bar{k}_1 = -28.89$, $\bar{k}_2 = 17.58$, $\bar{k}_3 = 3.68$. As $\hat{y}_1 = \lambda_1 \bar{k}_1 < 0$, we see that the disposable income is negative, which means negative consumption in country 1.

We evaluate the other variables at the unique equilibrium point, $k_1 = 13.58$ and $Z = 8.17$ as in Table 1. The global output is 34.18 and the interest rate is about 6.2 percent. The shares of the global outputs by the DE, ID and UD are respectively 15.20, 8.76 and 10.20 percent. It should be noted that the population shares of the three economies are respectively 20, 26.7 and 53.3 percent. The per-worker output levels of the DE, ID and UD are respectively 5.07, 2.19 and 1.28. The differences in labor productivity are mainly due to the differences in knowledge utilization efficiency. The table also gives the capital distribution among the three countries. The wage rates in the DE, ID and UD are respectively 3.55, 1.61 and 1.02.

Table 1 The Equilibrium Values of the Global Economy

Z	K	F	r	C
8.167	94.12	34.18	0.062	31.05

Country 1		Country 2		Country 3	
F_1	15.20	F_2	8.76	F_3	10.22
K_1	40.75	K_2	25.05	K_3	28.32
F_1 / N_1	5.07	F_2 / N_2	2.19	F_3 / N_3	1.28
k_1	13.58	k_2	6.26	k_3	3.54

$\bar{K}_1$	51.96	$\bar{K}_2$	21.74	$\bar{K}_3$	20.42
C_1	13.86	C_2	7.77	C_3	9.42
w_1	3.55	w_2	1.61	w_3	1.02
$\bar{k}_1$	17.32	$\bar{k}_2$	5.44	$\bar{k}_3$	2.55
$\hat{y}_1$	21.94	$\hat{y}_2$	7.34	$\hat{y}_3$	2.73
c_1	4.62	c_2	1.94	c_3	1.18

The trade balances of the three countries are given by

$$E_j(t) = \left(\bar{K}_j(t) - K_j(t)\right) r(t), \quad j = 1, 2, 3.$$

When $E_j(t)$ is positive (negative), we say that country j is in trade surplus (deficit). When $E_j(t)$ is zero, country $j's$ trade is in balance. We calculate the trade balances at equilibrium as follows

$$E_1 = 0.69, \quad E_2 = -0.21, \quad E_3 = -0.49.$$

The DE is in trade surplus and the other two economies in trade deficit.

So far we have been concerned with equilibrium. As the four eigenvalues at equilibrium are as follows

$$-0.267, \quad -0.214, \quad -0.174, \quad -0.025,$$

we see that the equilibrium is stable. We start with different initial states not far away from the equilibrium point and find that the system approaches to the equilibrium point. This implies that the system is stable. In Figure 2, we plot the motion of the system with the following initial conditions

$$k_1(0) = 17, \quad \bar{k}_2(0) = 8, \quad \bar{k}_3(0) = 3, \quad Z(0) = 17.$$

The system approaches to its equilibrium in the long term. It can be seen that convergence does not happen with the specified values of the parameters. In an international trade model with learning by doing and invention by Nakajima[2], catch-up is possible in the long run via different transition paths. In Nakajima's model, both learning by doing and invention are the engines of growth and countries have economic interactions through international trade. A poor country can catch-up rich ones as it has less constrains in the pace of invention than rich countries. In our model, this can happen, for instance, if poor countries improve its efficiency of knowledge utilization.

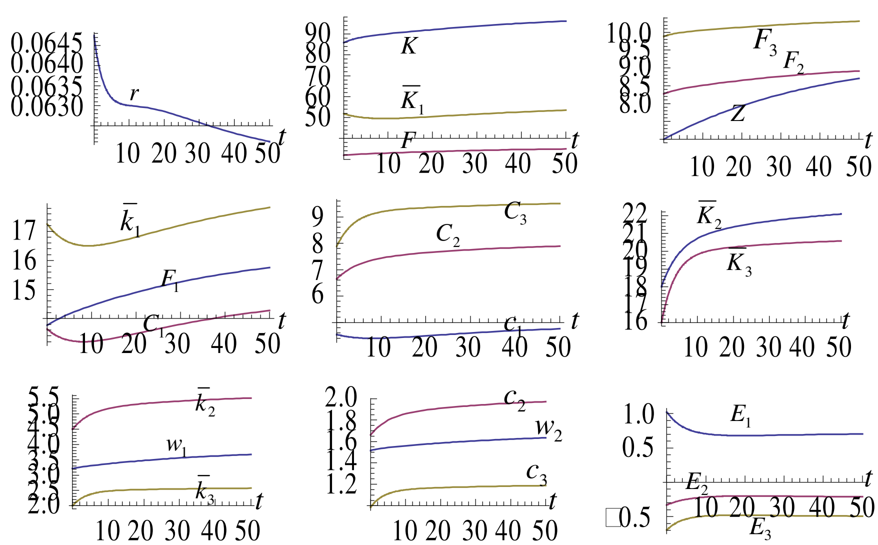

Figure 2 The Motion of Some Variables

4 Knowledge Utilization Efficiency and Global Growth

We simulated the motion of the dynamic system. It is important to ask questions such as how a developing economy like India or China may affect the global economy as its technology is improved or population is enlarged; or how the global trade patterns may be affected as technologies are further improved or propensities to save are increased in developed economies like the USA or Japan. The rest of this paper examines effects of changes in some parameters on dynamic processes of the global economic system. First, we examine the case that all the parameters, except country 1's knowledge utilization efficiency, m_1, are the same as in (16). We decrease the knowledge efficiency parameter, m_1, from 0.4 to 0.35. The simulation results are demonstrated in Figure 3. In the plots, a variable $\overline{\Delta} x_j(t)$ stands for the change rate of the variable, t in percentage due to changes in the parameter value from m_{10} ($= 0.4$ in this case) to m_1 ($= 0.35$). That is

$$\overline{\Delta} x_j(t) \equiv \frac{x_j(t;\, m_1) - x_j(t;\, m_{10})}{x_j(t;\, m_{10})} \times 100,$$

where $x_j(t;\, m_1)$ stands for the value of the variable x_j with the parameter value m_1 at time t and $x_j(t;\, m_{10})$ stands for the value of the variable x_j with the parameter value m_{10} at time t. We will use the symbol $\overline{\Delta}$ with the same meaning when we analyze other parameters.

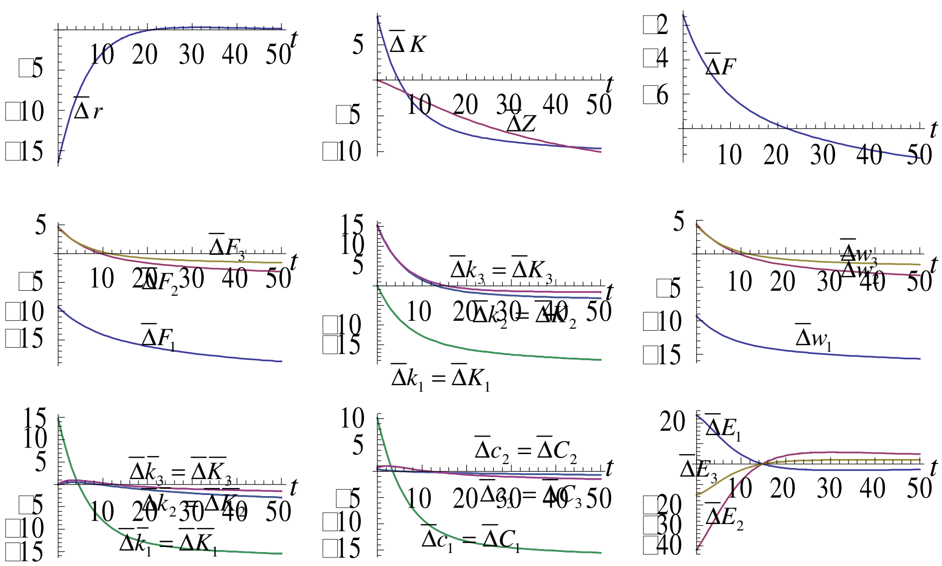

Figure 3 The Knowledge Utilization Efficiency Is Reduced in the Developing Economy

As the DE's knowledge utilization efficiency is reduced, the knowledge and capital of the global economy are reduced; the output level of the global economy falls. The DE's output level falls; the other two countries' output levels rise initially and then fall. As the rate of interest falls initially and knowledge rises but not much initially, we see that the costs of production are low for the IE and UE and their productivities are not much improved, the two economies' output levels rise initially. As time passes, the world accumulates less knowledge and the rate of interest rises, the IE's and UE's output levels are reduced. We see that in the long term the DE's trade balance is deteriorated and the other two economies' trade balances slightly improved. In the long term the wage rates and the levels of per capita consumptions and wealth in the three economies are all reduced. Hence, we conclude that as the UE's knowledge utilization efficiency falls, all the consumers in the globe suffer in the long term.

We now examine effects of the UE's knowledge efficiency upon the global economy. We allow: $m_3 : 0.1 \Rightarrow 0.15$. The effects of the UE's improvement in knowledge utilization are provided in Figure 4. The output level of the global economy rises all the time. The UE's output level rises; the other two countries' output levels are affected slightly. The rate of interest rises over the time. The DE and IE's trade balances are improved and the UE's trade balance deteriorates. In the long term the wage rate, the per-capita wealth and consumption level are increased; the wage rate, the per-capita wealth and consumption level of the DE and IE are affected slightly.

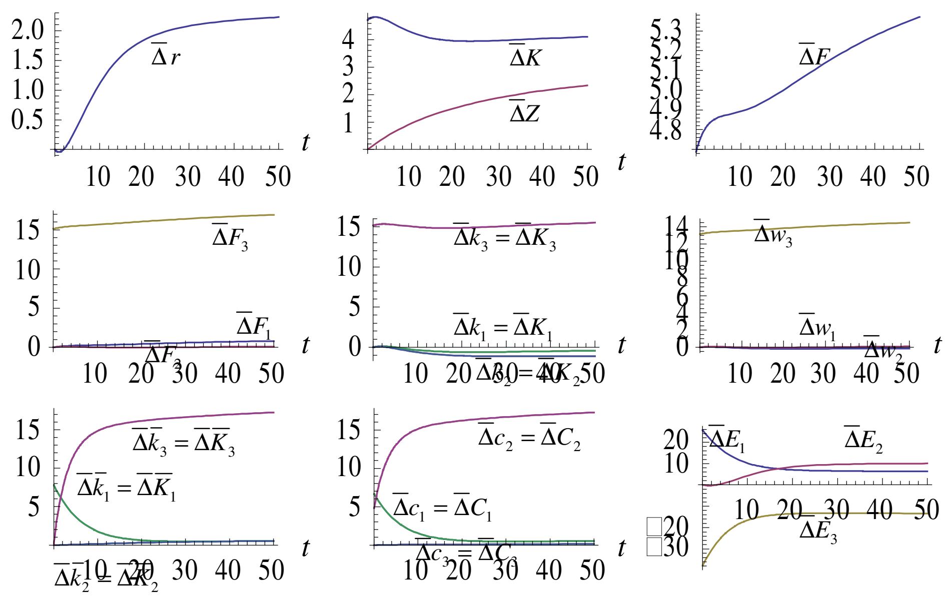

Figure 4 The Knowledge Utilization Efficiency Is Increased in the Developing Economy

5 Population Change and the Global Economy

The relationship between population change and economics is a challenging area. Although this study assumes the population fixed, it is important to examine effects of changes in the population sizes. As different countries have different levels of knowledge utilization efficiency and creativity, increases in the population sizes may have different effects upon the global economy. For instance, it is important to examine implications of possibly negative population growth in developed economies and rapid population growth in underdeveloped economies. It has been observed that the effect of population growth varies with the level of economic development and can be positive for some developed economies. Theoretical models with human capital predict situation-dependent interactions between population and economic growth[35-37]. First, we are concerned with the effects of an increase in the DE's population as follows: $N_1 : 3 \Rightarrow 3.2$. The effects are plotted in Figure 5. The knowledge, global wealth and output levels are increased. The rate of interest falls initially and then rises. The total output and consumption levels, total wealth, per capita consumption levels, and per capita wealth levels of the three economies are all increased in the long term. The trade balance of the DE improves and the other two economies deteriorate.

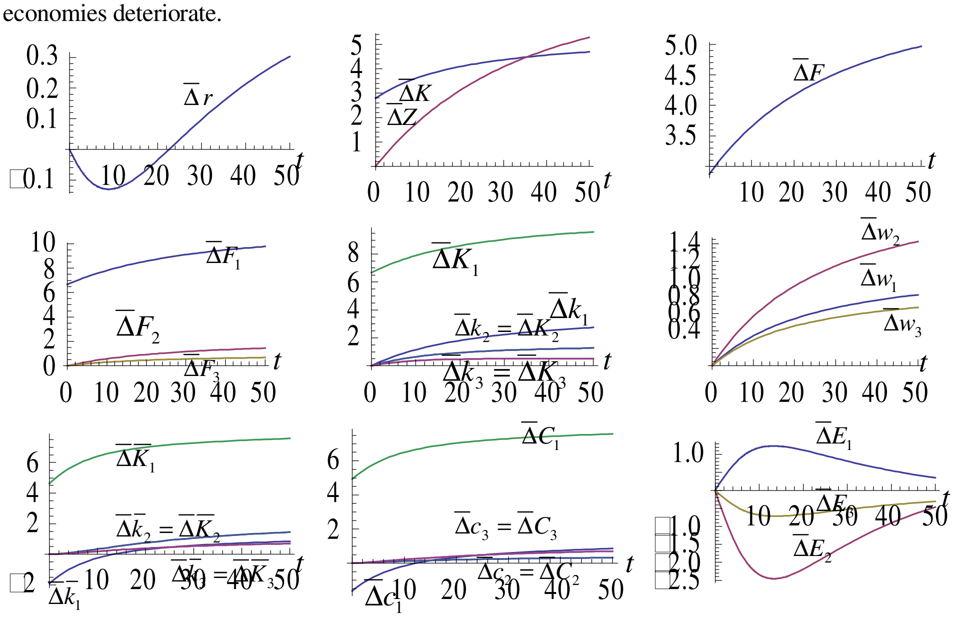

Figure 5 The Developed Economy Increases Its Population

We now examine the case when the DE's population changes as follows: $N_3 : 8 \Rightarrow 8.4$. The effects are plotted in Figure 6. The knowledge, global wealth and output levels are increased. The rate of interest rises. The output levels and total wealth of the UE and DE are all increased in the long term; but the output levels and total wealth of the UE will be reduced. The trade balance of the DE and IE improve and the trade balance of the UE deteriorates.

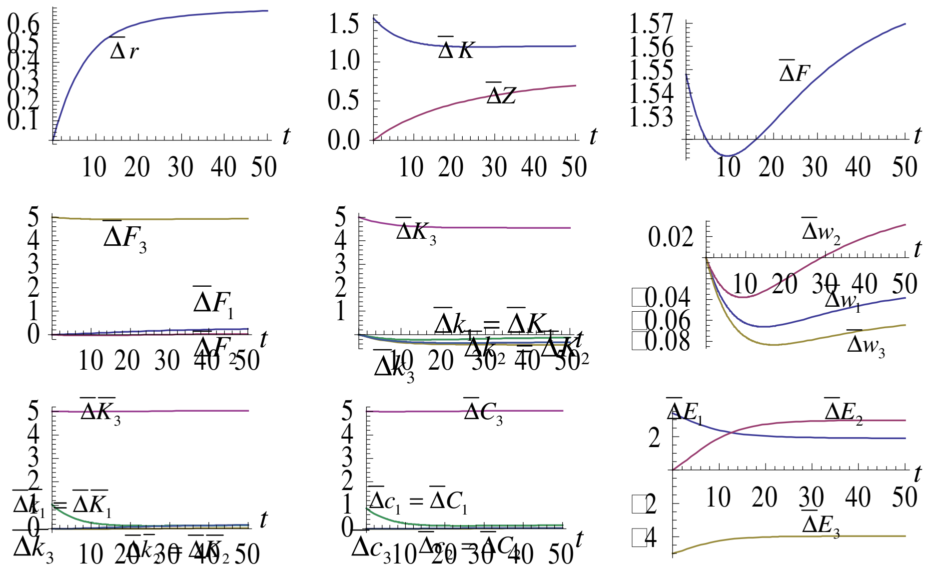

Figure 6 The Developing Economy Increases Its Population

6 Preference Change

Preference changes are important for explaining trade patterns with externalities and returns to scale. It should be noted Desdoigts and Jaramillo[38] have recently proposed a trade model to examine demand spillovers brought about by a global middle class. Their research interest is partly caused by possible effects of the emerging middle class in Brazil, Russia, India and China on global trade patterns and global economic growth. Our model also examines effects of preference changes, but emphasizing different aspects of consumer preferences. We now allow the DE to increase its propensity to save as follows: $\lambda_{01} : 0.75 \Rightarrow 0.77$. The results are plotted in Figure 7. As the DE increases its propensity to save, the knowledge, global wealth and output level are increased. The rate of interest is reduced. The output levels of the three economies are all increased. The wage rate, total consumption and wealth levels, per-capita wealth and consumption levels in the three economies are increased. The DE trade balance improves and the IE and UE's trade balance deteriorate. As the changes are measured in the change in percentage, we see that the gaps between the poor country and rich country are enlarged due to the preference change.

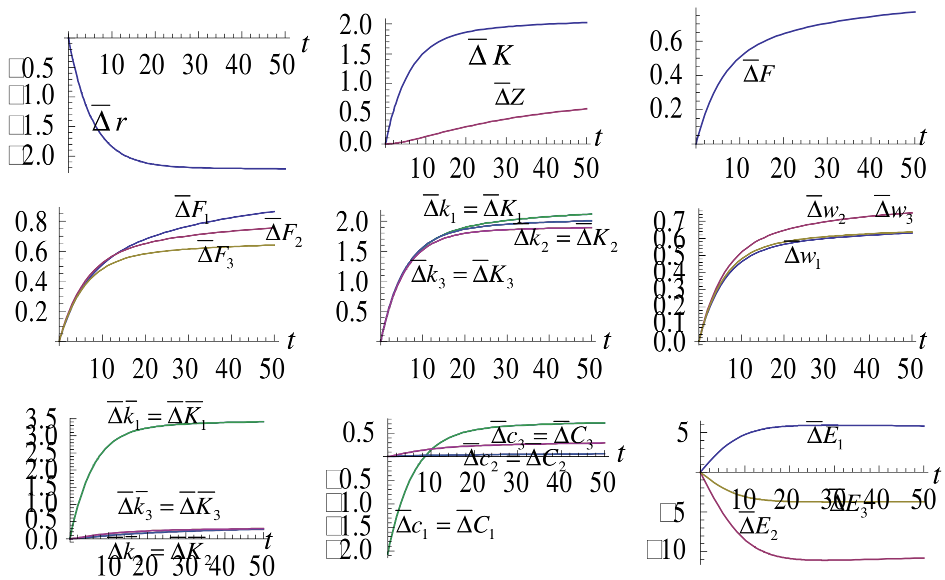

Figure 7 The Developed Economy Raises Its Propensity to Save

7 Conclusions

This paper proposed a multi-country growth model with capital accumulation and knowledge creation. Different from the growth models with the Ramsey approach in the literature, we used a utility function, which determines saving and consumption with utility optimization without leading to a higher dimensional dynamic system like by the traditional approach. We show that the dynamics of J-country world economy is controlled by a $(J+1)$-dimensional differential equations system. We also simulated the motion of the model and demonstrated effects of changes in the parameters. In our model, if an economy is effective in utilizing knowledge, it experiences fast growth and can maintain living standard at a high level. Our model does not generate global convergence. It is straightforward to demonstrate that if some economies exhibit increasing returns to scale, then there exist multiple equilibrium points. To which equilibrium point the system evolves to is dependent on initial conditions. It is well known that one-sector growth model has been generalized and extended in many directions. It is not difficult to generalize our model along these lines. It is straightforward to develop the model in discrete time. We may analyze behavior of the model with other forms of production or utility functions. There are multiple production sectors and households are not homogenous. In the contemporary literature, private research and endogenous population have been emphasized.

Appendix A1: Proving Lemma 1

First, from equations (1) we obtain

$$k_j = \phi_j(k_1, Z) \equiv \left(\frac{A_j \alpha_j Z^{m_j}}{A_1 \alpha_1 Z^{m_1} k_1^{-\beta_1} + \delta_j} \right)^{1/\beta_j}, \quad j = 1, \ldots, J, \tag{A1}$$

where $\delta_j \equiv \delta_{k1} - \delta_{kj}$. It should be noted that $\phi_1 = k_1$. From equations (1) and (A1), we determine the wage rates as functions of $k_1(t)$ and $Z(t)$ as follows

$$w_j = \bar{\phi}_j(k_1, Z) \equiv A_j \beta_j Z^{m_j} \phi_j^{\alpha_j}(k_1, Z), \quad j = 1, \cdots, J. \tag{A2}$$

By $K_j(t) = k_j(t) N_j$, $K_j(t)$ are also functions of $k_1(t)$ and $Z(t)$. We see that the capital distribution among the countries are uniquely determined as functions of $k_1(t)$ and $Z(t)$. By $K = \sum_{j=1}^{J} K_j$, we see that K is also uniquely determined as a function of k_1 and Z. We denote this function as follows

$$K = \psi(k_1, Z).$$

Substituting $F_j = Z^{m_j} K_j^{\alpha_j} N_j^{\beta_j}$ into equation (8), we have

$$\dot{Z} = \Lambda(k_1, Z) \equiv \sum_{j=1}^{J} \tau_j A_j N_j^{\beta_j} Z^{m_j - \varepsilon_j} K_j^{\alpha_j} - \delta_z Z. \tag{A3}$$

We see that the motion of Z can be described as a unique function of k_1 and Z.

From equations (12), we solve

$$\bar{k}_1 = n_0 \psi(k_1, Z) - \sum_{j=2}^{J} n_j \bar{k}_j, \tag{A4}$$

in which

$$n_0 \equiv \frac{1}{N_1}, \quad n_j \equiv \frac{N_j}{N_1}, \quad j = 2, ..., J.$$

Introduce $\{\bar{k}(t)\} \equiv (\bar{k}_2(t), \cdots, \bar{k}_J(t))$. We see that country 1's per capita wealth, $\bar{k}_1(t)$, can be expressed as a unique function of the knowledge, country 1's capital intensity and the other countries' per capita wealth, $\{\bar{k}(t)\}$, at any point of time.

From equations (2) and (3), we have

$$\hat{y}_j = (1 + r)\bar{k}_j + w_j. \tag{A5}$$

Substituting $s_j = \lambda_j \hat{y}_j$ and the above equations into equations (7), we have

$$\dot{\bar{k}}_1 = \lambda_1 w_1 - R(k_1, Z)\bar{k}_1, \tag{A6}$$
$$\dot{\bar{k}}_j = \Lambda_j(k_1, \bar{k}_j, Z) \equiv \lambda_j w_j - (1 - \lambda_j - \lambda_j r)\bar{k}_j, \quad j = 2, ..., J, \tag{A7}$$

in which $R(k_1, Z) \equiv 1 - \lambda_1 - \lambda_1 r$. Equations (A7) are the differential equations for $\bar{k}_j(t)$ in Lemma 2, $j = 2, ..., J$. Taking derivatives of equation (A7) with respect to t yields

$$\dot{\bar{k}}_1 = n_0 \psi_{k_1} \dot{k}_1 + n_0 \psi_Z \dot{Z} - \sum_{j=2}^{J} n_j \dot{\bar{k}}_j, \tag{A8}$$

where ψ_{k_1} and ψ_Z are the partial derivatives of $\psi(k_1, Z)$ with respect to k_1 and Z. Equaling the right-hand sizes of equations (A6) and (A8), we get

$$n_0 \psi_{k_1} \dot{k}_1 + n_0 \psi_Z \dot{Z} - \sum_{j=2}^{J} n_j \dot{\bar{k}}_j = \lambda_1 w_1 - R\bar{k}_1.$$

Substitute equation (A7) into the above equation

$$\dot{k}_1 = \Lambda_1(k_1, \{\bar{k}_j\}, Z) \equiv \left[\sum_{j=2}^{J} n_j \Lambda_j + \lambda_1 w_1 - n_0 R \psi + R \sum_{j=2}^{J} n_j \bar{k}_j - n_0 \psi_Z \Lambda\right] \frac{1}{n_0 \psi_{k_1}}, \tag{A9}$$

where we use equations (A4) and (A3). This is the differential equation for $k_1(t)$ in Lemma 1. Substitute equations (A1) into equation (A3), we have

$$\dot{Z} = \Lambda(k_1, Z) = \sum_{j=1}^{J} \bar{\tau}_j Z^{m_j - \varepsilon_j} \phi_j^{\alpha_j} - \delta_z Z,$$

where $\bar{\tau}_j \equiv \tau_j A_j N_j$. This is the differential equation for $Z(t)$ in Lemma 1. In summary, we have proved Lemma 1.

Appendix A2: Global Dynamics with the Identical Preference

We now examine a special case when the households in the world have the identical preference and the depreciation rates are the same among the economies. It should be noted that Bougheas and Riezman[39] examine trade pattern where the only difference between the two countries is in their distribution of human capital endowments. Their consumers' preferences are identical. After certain reformation of the economic structure, our model can deal the issue in their model if we interpret the knowledge utilization is human capital. An interesting direction in their model for extending our model is that the model introduces a majority voting framework to examine relations between trade and politics. We are interested in this case as because the dynamic analysis becomes simpler. We require

$$\xi = \xi_j, \quad \lambda = \lambda_j, \quad \delta_k = \delta_{kj}, \quad \alpha = \alpha_j, \quad j = 1, ..., J.$$

We now show that all the variables in the dynamic system can be expressed as functions of $k_1(t)$ and $Z(t)$ at any point. First, from equations (1) we obtain

$$k_j = M_j Z^{\bar{m}_j} k_1, \quad j = 1, ..., J, \tag{A10}$$

in which

$$M_j \equiv \left(\frac{A_j}{A_1}\right)^{1/\beta}, \quad \bar{m}_j \equiv \frac{m_j - m_1}{\beta}.$$

Country $j's$ capital intensity of the production function can be expressed as a unique function of the knowledge and country 1's capital intensity of the production function. We determine the rate of interest and the wage rates as functions of $k_1(t)$ and $Z(t)$ as follows

$$r = A_1 \alpha Z^{m_1} k_1^{-\beta} - \delta_k, \quad w_j = A_j \beta M_j^{\alpha} Z^{\alpha \bar{m}_j + m_j} k_1^{\alpha}, \quad j = 1, \cdots, J. \tag{A11}$$

By $k_j = K_j / N_j$ and equations (A10), we have

$$K_j = \bar{M}_j Z^{\bar{m}_j} k_1, \quad j = 1,..., J, \tag{A12}$$

where $\bar{M}_j = N_j M_j / N_1$. Adding all the equations in (A12) yields

$$K = k_1 \Lambda_0(Z), \tag{A13}$$

where we use $K = \sum_{j=1}^{J} K_j$ and

$$\Lambda_0(Z) \equiv \sum_{j=1}^{J} \bar{M}_j Z^{\bar{m}_j}.$$

From $F_j = A_j Z^{m_j} K_j^{\alpha} N_j^{\beta}$ and equations (A12), we have

$$F_j = A_j N_j^{\beta} \bar{M}_j^{\alpha} Z^{m_j + \alpha \bar{m}_j} k_1^{\alpha}. \tag{A14}$$

Substituting equations (A14) into equation (8), we have

$$\dot{Z} = \Lambda(k_1, Z) \equiv \sum_{j=1}^{J} \tau_j A_j N_j^{\beta} \overline{M}_j^{\alpha} Z^{m_j + \alpha \overline{m}_j - \varepsilon_j} k_1^{\alpha} - \delta_z Z. \tag{A15}$$

We see that the motion of Z can be described as a unique function of k_1 and Z. From equations (2) and (3), we have $\hat{y}_j = (1+r)\bar{k}_j + w_j$. Substituting $s_j = \lambda \hat{y}_j$ and the above equations into equations (7), we have

$$\dot{\bar{k}}_j = \lambda w_j - (1 - \lambda - \lambda r)\bar{k}_j. \tag{A16}$$

Multiplying the equation for $\bar{k}_j$ by N_j and then adding the J resulted equations, we have

$$\dot{K} = \lambda \beta k_1^{\alpha} \sum_{j=1}^{J} A_j M_j^{\alpha} Z^{\alpha \overline{m}_j + m_j} - \left(\bar{\lambda} - A_1 \alpha \lambda Z^{m_1} k_1^{-\beta}\right) K, \tag{A17}$$

where we use equations (A14) and $K = \sum_{j=1}^{J} \bar{k}_j N_j$ and $\bar{\lambda} \equiv 1 - \lambda + \lambda \delta_k$. Taking derivatives of equation (A13) with respect to t yields

$$\dot{K} = \Lambda_0(k_1, Z) \equiv \frac{K \dot{k}_1}{k_1} + \left(k_1 \sum_{j=1}^{J} \overline{m}_1 \overline{M}_j Z^{\overline{m}_j - 1}\right) \dot{Z}. \tag{A18}$$

Substituting equations (A17) and (A13) into equation (A18) yields

$$\dot{k}_1 = \frac{\lambda \beta k_1^{\alpha}}{\Lambda_0} \sum_{j=1}^{J} A_j M_j^{\alpha} Z^{\alpha \overline{m}_j + m_j} - \left(k_1 \sum_{j=1}^{J} \overline{m}_1 \overline{M}_j Z^{\overline{m}_j - 1}\right) \frac{\Lambda}{\Lambda_0} - \left(\bar{\lambda} - A_1 \alpha \lambda Z^{m_1} k_1^{-\beta}\right) k_1. \tag{A19}$$

Summarizing the above results, we obtain the following lemma.

Lemma A1

Assume that all the households in the world have the same preference. The motion of the two variables, $k_1(t)$ and $Z(t)$, are given by two differential equations, (A15) and (A19). For any given $k_1(t)$ and $Z(t)$, we determine $r(t)$ and $w_j(t)$, $j = 1, \cdots, J$, by (A11). The variables, $\bar{k}_j(t)$, are solved by equations (A16) as follows

$$\bar{k}_j(t) = e^{-\int (1 - \lambda - \lambda r) d\tau} \left(h_j + \lambda \int w_j(\tau) e^{\int (1 - \lambda - \lambda r) d\tau} d\tau\right), \quad j = 1, \cdots, J, \tag{A20}$$

where h_j are constants to be determined by initial conditions. For any given positive values of $Z(t)$, $k_1(t)$ and $\bar{k}_j(t)$ at any point of time, the other variables are uniquely determined by the following procedure: $k_j(t)$, $j = 2, \cdots, J$ by (A10) $\rightarrow$ $K(t)$ by (A18) $\rightarrow$ $K_j(t) = N_j k_j(t)$ $\rightarrow$ $\hat{y}_j = (1+r)\bar{k}_j + w_j$ $\rightarrow$ $c_j(t)$ and $s_j(t)$ by (6) $\rightarrow$ $F_j = Z^{m_j} K_j^{\alpha} N_j^{\beta}$.

The dynamic properties of the world economy are determined by two differential equations. Equilibrium is determined by

$$\sum_{j=1}^{J} \tau_j A_j N_j^{\beta} \overline{M}_j^{\alpha} Z^{m_j + \alpha \overline{m}_j - \varepsilon_j} k_1^{\alpha} = \delta_z Z,$$

$$\frac{\lambda \beta k_1^{\alpha}}{\Lambda_0} \sum_{j=1}^{J} A_j M_j^{\alpha} Z^{\alpha \overline{m}_j + m_j} - \left(\bar{\lambda} - A_1 \alpha \lambda Z^{m_1} k_1^{-\beta}\right) k_1 = 0. \tag{A21}$$

By the second equation in equations (A21), we solve

$$k_1 = \Omega_0^{1/\beta}\left(\frac{\lambda}{\bar{\lambda}}\right)^{1/\beta}, \qquad \text{(A22)}$$

where

$$\Omega_0(Z) \equiv \frac{\beta}{\Lambda_0}\sum_{j=1}^{J} A_j\, M_j^{\alpha}\, Z^{\alpha\bar{m}_j + m_j} + A_1\,\alpha Z^{m_1}.$$

Substitute equation (A22) into the first equation in equations (A21)

$$\Omega(Z) \equiv \sum_{j=1}^{J} \tau_j\, A_j \left(\frac{\lambda}{\bar{\lambda}}\right)^{\alpha/\beta} N_j^{\beta}\, \bar{M}_j^{\alpha}\, Z^{m_j + \alpha\bar{m}_j - \varepsilon_j - 1}\, \Omega_0^{\alpha/\beta} - \delta_z = 0. \qquad \text{(A23)}$$

From Lemma A and the above discussions, we have the following corollary.

Corollary A1
The number of equilibrium points is the same as the number of solutions of $\Omega(Z) = 0$, for $Z > 0$. For any solution $Z > 0$, all the other variables are uniquely determined by the following procedure: k_1 by (A22) $\rightarrow$ r and w_j, $j = 1, \cdots, J$, by (A14) $\rightarrow$ $\bar{k}_j = \lambda w_j /(1 - \lambda - \lambda r)$ $\rightarrow$ k_j, $j = 2, \cdots, J$ by (A10) $\rightarrow$ K by (A18) $\rightarrow$ $K_j = N_j k_j$ $\rightarrow$ $\hat{y}_j = (1 + r)\bar{k}_j + w_j$ $\rightarrow$ c_j and s_j by (6) $\rightarrow$ $F_j = Z^{m_j} K_j^{\alpha} N_j^{\beta}$.

The number of equilibrium points is the same as the number of solutions of $\Omega(Z) = 0$, for $Z > 0$. As the expression is tedious, it is difficult to explicitly judge under what conditions the equation has a unique or multiple equilibrium points.

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