Applying multilevel confirmatory factor analysis techniques to perceived homework quality

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Abstract

This research study aimed to propose the multilevel confirmatory factor analysis. The study also focused on the multilevel confirmatory factor analysis of students' perceived homework quality via 4 indicators: 1) homework content, 2) homework explanation, 3) homework check, and 4) homework discussion. The subjects were 1427 students from five campuses of Rajabhat University, drawn from 40 classes. The proposed multilevel confirmatory factory model of homework quality fit well with the empirical data set ($\chi^2=2.084$, $\text{df}=2$, $\chi^2/\text{df}=1.042$, $p\text{-value}=0.3527$, $\text{CFI}=1.000$, $\text{TLI}=1.000$, $\text{RMSEA}=0.005$, $\text{SRMR}_W=0.006$, $\text{SRMR}_B=0.018$). The coefficient of determination of the student-level effects was 0.37-0.53 and the coefficient of determination of the classroom-level effects was 0.56 - 0.90.

Keywords: multilevel confirmatory factor analysis (MCFA), homework
Introduction

The attitude of students at all levels towards their homework assignments has been one of the most popular topics of discussion among teachers, parents, and educators (Simplicio, 2005; Marzano, 2007). The syntheses of research conducted by Cooper (1989), and Cooper, Robinson, & Patall (2006) showed that doing homework helped increase students' learning, and enhance students' self-discipline in managing the completion of their homework before the due date. However, the research studies on homework during the period of 1987 to 2006, regardless of their types, had design flaws. The researchers used homework as a learning and teaching tool, and only assigned the experimental groups homework, while the control groups were not given any homework assignments (Kohn, 2006; Cooper, Robinson, & Patall, 2006). Homework is complex because there are different groups of people, e.g. teachers, students, and parents, involved. Also, it serves a variety of purposes, e.g. achievement, improvement, self-regulation; engages tasks of different quality levels e.g. routine tasks versus complex tasks, and affects lesson organization, e.g. discussing, checking, and grading homework. Therefore, research studies on homework should incorporate new methodologies, such as multilevel modeling so that homework-related research studies will be put on the right track (Trautwein & Koller, 2003).

According to Trautwein et al. (2006a), researchers have provided some guidance of how to conduct research studies on homework by using the Multilevel Homework Model, which combines elements of expectancy-value theory (Eccles & Wigfield, 2002; Wigfield & Eccles, 2000), research on learning and instruction (Weinert & Helmke, 1995), and self-determination theory (Deci & Ryan, 2002). Stable personal characteristics, namely basic cognitive abilities and conscientiousness, (Costa & McCrae, 1992) are also included. In many studies, researchers used homework as a basic example of problems between teachers and students that affected students' achievement in their studies. Therefore, it is important for all studies to relate homework to students' success in order to look at its effects at the classroom-level and the student-level (Trautwein et al., 2002; Trautwein & Koller, 2003; Trautwein et al., 2006a; Trautwein et al., 2006b; Trautwein & Ludtke, 2007; Trautwein, 2007; Trautwein & Ludtke 2009).

The multilevel analysis can solve the technical problems of the conventional method in the areas of aggregation bias, misestimated standard error and heterogeneity of regression, but it does not give importance to the causal structural relationship between variables (Raudenbush & Bryk, 2002; Farmer, 2000). The Structural Equation Model (SEM), on the other hand, was created to show the relationship between latent variables, and between latent variables and observed variables (Diamantopoulos & Siguaw, 2000). However, its limitation lies in its lack of focus on the natural structure of hierarchical data (Muthén, 1994). The multilevel analysis and Structural Equation Model have been developed into the Multilevel Structural Equation Model that can analyze the relationship between hierarchical latent variables. This technique is then suitable for the analysis of homework-related variables that are multilevel and complex. This can solve the weaknesses of the traditional techniques.

In this research study, the researchers, then, proposed a Multilevel Confirmatory Factor Analysis Model of students' perceived homework quality in the business statistics course.

1. Methodology

1.1 Sample

The sample group comprised undergraduate students in the business statistics course from the faculty of Business Management, Rajabhat University. The Simple Random
Sampling technique was used to select the sample group from the population. Five out of nine Rajabhat University's central region campuses were chosen. They included 40 classes with the average of homework effort scores of 35.68. There were more than 18 students enrolling in each class. The total number of the students who participated in the study was 1427. This corresponded with the rule requiring that the number of the students in the sample group be larger than the number of the studied variables (Muthén, 1989) and the number of groups recommended was about 20 to 100 (Hox & Mass, 2001; Hox & Kreft, 1994; Hox, 1993).

1.2 Data collection

The researcher contacted the instructors of the statistics course at each campus and collected the data by distributing a questionnaire to the students to complete. The time allowed to answer the questions on the questionnaire was limited to 20 minutes.

1.3 Instrument

The instrument that was used in this study was a 5-point Likert Scale questionnaire. It tested students’ homework quality perceptions in the statistics course. There were 4 observed variables incorporated in the questionnaire: 1) homework content, 2) homework explanation, 3) homework discussion, and 4) homework feedback. Nineteen questions were created and modified based on the work of Trautwein, Ludtke, Schnyder, et al. (2006). The coefficient of determination of the student-level effects was homework content(R2=0.53), homework explanation (R2=0.53), homework check(R2=0.37), and homework discussion(R2=0.44). The coefficient of determination of the classroom-level effects was homework content(R2=0.79), homework explanation(R2=0.85), homework check(R2=0.56) and homework discussion(R2=0.90).

1.3 Statistical analyses: Analyzing multilevel confirmatory factor analysis procedures

Multilevel confirmatory factor analysis (MCFA), a multilevel SEM technique, was originally devised to test the factor structure of responses to a measurement instrument used in a study by means of which participants can be categorized into different groups (e.g., Hox, 1998; Zimprich, Perren, & Hornung, 2005; Sun & Willson, 2008). Multilevel confirmatory factor analysis model may be described as combining one separate factor analysis model which accounts for the structure of observations on individuals within groups, and another factor analysis model which accounts for the structure of observed group means. Multilevel model thus implies a covariance structure model that is formulated in terms of a conventional factor analysis model on both “between-group” and “within-group” levels. (Muthén, 1989, 1994) MCFA should involve five steps: (a) conventional confirmatory factor analysis (CFA), (b) intraclass correlation calculation, (c) within-group factor structure (d) between-group factor structure, and (e) MCFA. (Muthén, 1994)

Muthén (1994 cited in Dyer G. N. et al., 2005) developed the MCFA procedure. Figure 1 illustrates two – level confirmatory model with three observed indicators (y1W – y3W) depicted by squares. These indicators are the observed respondent ratings for the three items in a scale. The lower half of figure 1, labeled “within”, is consistent with a traditional confirmatory factor analysis on disaggregate data. As shown in this figure, the three observed variables load onto a single latent factor (ηW) at the “within” level. There are also three random errors (ε1W - ε3W) associated with each item at this level. The upper half of figure 1, labeled “between”, shows three indicators represented by the circled y1B - y3B. These are not observed/raw data, but rather represent the group means for each observed indicator (y1W –
y_{3w}). These group means load onto the aggregate latent variable (\(\eta_B\)) and are associated with their respective random error terms (\(\varepsilon_{1B} - \varepsilon_{3B}\)). The full model connects the disaggregate and corresponding aggregate indicators. Thus, the observed values of the original indicators (\(y_{1w} - y_{3w}\)) are considered to be a function of both the within and between-level latent constructs (\(\eta_W\) and \(\eta_B\), respectively). The two-level confirmatory model consists of a simultaneous analysis of both of the within and between-group covariance matrices.

- **Between**
  - \(\eta_B\)
  - \(\varepsilon_{1B}\)
  - \(\varepsilon_{2B}\)
  - \(\varepsilon_{3B}\)
  - \(y_{1B}\)
  - \(y_{2B}\)
  - \(y_{3B}\)

- **Within**
  - \(\eta_W\)
  - \(\varepsilon_{1w}\)
  - \(\varepsilon_{2w}\)
  - \(\varepsilon_{3w}\)
  - \(y_{1w}\)
  - \(y_{2w}\)
  - \(y_{3w}\)

**Figure 1 Multilevel confirmatory factor analysis model**

In figure 1, the between and within components are explained by a single latent factor, however, this need not be the case. For example, one could test a model that proposes a single factor at the aggregate level and two factors at the disaggregate level, or many other similar non-isomorphic structures. If the hypothesized factor structure proposes more than one factor at a given level, the model may also include covariances among those same-level factors (by definition in this type of model, no covariances are allowed among factors at different levels). Similarly, the model may suggest that some indicators are valid at one level only, indicating a fuzzy composition model. Furthermore, the model may show some important covariates (e.g., age, pretest) that might be included in the model, relate to the focal latent construct at only one level. Estimation of these models yields both indicators of model fit, and parameter estimates of the factor loadings, factor variances, and uniquenesses (residuals). Thus, although our illustration presents only a very simple case, the MCFA technique in general promises some flexibility in the type of model that can be specified and tested. An advantage of the MCFA is that the individual- and class-level factor structures are calculated in one step by separating the total covariance into two parts - one between groups and one within groups (i.e. individuals; e.g. Mathisen et al., 2006; McDonald,1993; Muthén, 1991).

Six indices were used to assess the measurement model’s fit to the data with the MCFA. These indices included the \(\chi^2\) index, the goodness-of-fit index (GFI), the nonnormed fit index (NNFI), the comparative fit index (CFI), the root mean square error of approximation (RMSEA), and the standardized root mean residual (SRMR). The MCFA models were tested with Muthén’s maximum likelihood (MUMLL), which includes robust standard errors and adjustment to the \(\chi^2\) test statistic due to unbalanced group sizes. MUMLL procedure leads to correct model inference asymptotically when level-2 sample size goes to infinity and the coefficient of variation of the level-1 sample sizes goes to zero (Yuan H. K. & Hayashi K., 2005). The six above-mentioned fit indices were chosen for this study because...
no single fit index is considered to be the definitive marker of a model with “good” fit; each index serves a different purpose and should be interpreted in combination with the other indices. The $\chi^2$ index is an absolute index that tests for lack of fit resulting from overidentifying restrictions placed on a model. A nonsignificant $p$ value (e.g., $p > 0.05$) is desired, but the $\chi^2$ index is usually inflated by the number of restrictions imposed on a model and sample size. Values of 1 for the GFI and the NNFI indicate perfect model fit; however, some researchers have suggested cutoff values greater than 0.95 to indicate good model fit. The following fit index cutoff values suggested by Hu and Bentler (1999) were used for determining goodness of fit: $\text{CFI} > 0.95$, $\text{RMSEA} < 0.06$, and $\text{SRMR} < 0.08$.

1.5 Missing values

We analyzed using a special feature of Mplus, has several options for the estimation of models with missing data. Mplus provides maximum likelihood estimation under MCAR (missing completely at random) and MAR (missing at random; Little & Rubin, 2002) for continuous, censored, binary, ordered categorical (ordinal), unordered categorical (nominal), counts, or combinations of these variable types. (Muthén & Muthén, 2007)

2. Results

2.1 Conventional confirmatory factor analysis: Step 1

An a priori one-factor model with paths from the latent construct to all four homework quality items was tested by using the total sample matrix. Model fit indices are $\chi^2 / df = 1.06, p < 0.01$, $\text{CFI} = 1.000$, $\text{TLI} = 1.000$, $\text{RMSEA} = 0.007$, and $\text{SRMR} = 0.024$. The result of the confirmatory factor analysis’s homework quality (see in Table 1) showed that the multilevel confirmatory factor homework effort model had structural validity, or fit the empirical data but was not extremely, although the values of the CFI, the RMSEA, and the SRMR were in range suggestion adequate fit because this model ignores the nested data structure.

Table 1 Model fit for priori single -and multilevel models

<table>
<thead>
<tr>
<th>Model</th>
<th>df</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
<th>SRMR</th>
<th>W:0.018</th>
<th>B:0.015</th>
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<td>1.00</td>
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<tr>
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<td>1.00</td>
<td>0.00</td>
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<td>0.006</td>
<td>0.018</td>
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Table 1 Model fit for priori single -and multilevel models

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Note. Average cluster size (c) = 35.68, $|Z| > 2.58$; $p < .01$, $\chi^2 / df = 1.042$, $p$-value=0.3527

Chi-square values are statistically significant at $p<0.01$. df=degrees of freedom, CFI=comparative fit index, RMSEA=root mean square error of approximation, SRMR=standardized root mean square residual. W=within-group portion of the model, B=between-group portion of the model.
2.2 Intraclass correlation: Step 2

The analysis of the elements of Multilevel Confirmatory Factor Analysis requires two-level variance. Intraclass correlation (ICC) is used to test whether the variables at the student level show variance only within groups, or also between groups or at the classroom level. If the ICC is more than 0.05, it means there are high correlations among variables, suitable to be tested by means of Multilevel Factor Analysis. However, if the ICC is less than 0.05, this means there is no variance at the classroom level. It is, therefore, not necessary to evaluate the data by using Multilevel Factor Analysis. Snijders and Bosker (1999) suggested that the ICC value should be more than 0.05, and based on Table 1, the ICC value of each observed variable ranged between 0.052 and 0.129. This showed that it was appropriate to use the Multilevel Factor Analysis with this set of data.

2.3 Within-group factor structure and between-group factor structure: Step 3-4

A student-level CFA model was tested by using the covariance matrix (SPW) based on individual-level scored. Model fit indices are $\chi^2$/df=1.438, p<0.01, CFI=1.000, TLI=0.999, RMSEA=0.018 and SRMR=0.010 (see in Table 1). A classroom-level CFA model was tested by using the between-group population matrix. Model fit indices are $\chi^2$/df=0.0171, p<0.01, CFI=1.000, TLI=1.000, RMSEA=0.000 and SRMR=0.005 (see in Table 1).

A classroom-level CFA analysis’s homework quality fit the empirical data well at the between–group factor structure, and adequate, but slightly worse fit at a student-level CFA model as indicated by the SRMR of 0.005.

2.4 Multilevel confirmatory factor analysis’s homework quality result: Step 5

The result of the multilevel confirmatory factor analysis’s homework quality showed that the multilevel confirmatory factor homework quality model had structural validity. Model fit indices are $\chi^2$/df=1.042, p<0.01, CFI=1.000, TLI=1.000, RMSEA=0.005, SRMR_B=0.006 and SRMR_W=0.018 (see in Table 1). The multilevel confirmatory factor analysis’s homework quality fit the empirical data well at the between level, and adequate, but slightly worse fit at student level as indicated by the SRMR of 0.006.

The intercepts or the average group mean were between 3.48 to 3.88. This showed that at the classroom level, intercepts of student perceptions of homework quality as shown by each variable ranged from medium to high levels. The variable “explanation” gained the highest intercepts of 3.88, while “discussion” obtained the least intercepts value of 3.48. Parameter estimates from this model included factor loadings at both the within and between level, as can be seen in Table 1. The items load strongly onto the single factor at the between level, ranging from 0.75 (check) to 0.95 (discussion). The factor loadings of the items at the within level, ranging from 0.61 (check) to 0.73 (explanation), are not as strong as the between factor loadings (see Figure 2).

Regarding the variance of students’ homework quality perceptions, which was considered the latent variable in this study, the Coefficient of Determination ($R^2$) at the student level was 0.612-0.727 and at classroom level was 0.750-0.949 (see table 1). This showed that the four observed variables could explain the covariance of homework quality at the student level at the percentage of 61.2-72.7 and at classroom level at the percentage of 75.0-94.9. The Coefficient of Determination ($R^2$) value implied that homework quality could explain the variance at the classroom level better than that at the student level.
To conclude, the multilevel confirmatory factor homework quality model that included four observed variables, which were homework content, homework explanation, homework discussion, and homework feedback possessed structural validity at both the student and the classroom levels.

![Multilevel confirmatory factor homework quality model](image)

**Figure 2 Multilevel confirmatory factor homework quality model**

### 3. Discussion

This research study was to develop and validate the multilevel homework quality model through 4 indicators. The research study revealed that students’ homework quality perceptions in the business statistics course showed variance at both the student and the classroom levels. The data appropriate to be analyzed by Multilevel Confirmatory Factor Analysis. This corresponded with students’ homework quality perceptions studies that also illustrated variance of homework quality perceptions at both levels (Trautwein et al., 2006; Luedtke et al., 2007; Trautwein & Lüdtke, 2009). Homework-related research studies should pay careful attention to the data with two-level variance to avoid incorrect research conclusions, since the variables related to homework are, by nature, multilevel and hierarchical nested data.

The intercepts or the average group mean were between 3.48 to 3.88. This showed that at the classroom level, intercepts of student perceptions of homework quality as shown by each variable ranged from medium to high levels. The variable “explanation” gained the highest intercepts of 3.88, while “discussion” obtained the least intercepts value of 3.48. Therefore, instructors should have discussed homework within classroom. This would also be beneficial for the students because what they learned from the business statistics course is considered basic knowledge of other courses.

Multilevel confirmatory factor analysis that was used to validate the multilevel homework quality model revealed that the model possessed structural validity or perfectly fit the empirical data. It was able to confirm that the variable “homework quality” could be used with the multilevel model, and the factor loading value of the student level was less than that
of the classroom level. At the student level, the observed variables that gained the highest value were explanation and content, followed by discussion and check respectively. Regarding the classroom level, the observed variables that gained the highest value were discussion, followed by explanation content and check respectively.

As for the variables' ability to explain variance, at the student level the four variables could explain the latent variable “homework quality” at the percentage of 37.4-52.8 whereas the number ranged between 56.3 and 90.0 at the classroom level. This meant that at the student level, the observed variable that could best explain the latent variable “homework quality” was explanation, at the classroom level was discussion. In addition, the observed variables could explain the latent variable “homework quality” better at the classroom level than at the student level.

To conclude, the validation of the multilevel confirmatory factor homework quality model confirmed that the model that incorporated the four observed variables had structural validity and could be analyzed at the two levels. The multilevel confirmatory factor analysis not only tests influence factor structure between group, but also facilitates the testing of theoretical hypotheses at different levels and has substantial potential for helping homework researches. We hope that this paper will lead to a more widespread use the multilevel confirmatory factor model (MCFA) in homework variables.

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References


