

## **A research framework for studying conceptions and dispositions of mathematics: A dialogue to help students learn**

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### **Abstract**

The paper discusses the research framework used in a study investigating student and teacher conceptions and dispositions of mathematics. Conceptions of mathematics refer to views that students hold of mathematics, and what they believe is required in learning and doing it. Dispositions of mathematics refer to beliefs or tendencies to exhibit a frequent, conscious and voluntary behavior directed towards learning mathematics. The research framework for this study is depicted by two nonintersecting circles, one representing conceptions and one representing dispositions of mathematics. Each circle has six equal portions that represent indicators of a conception or a disposition. The authors suggest that the six indicators of disposition represent the pre-conditions necessary for the interaction between dispositions and their conceptions of mathematics. This interaction will be explored in the research study.

The research study utilizes mixed methods with a triangulation of data through the use of a survey design, interviews and focus groups. An exploratory questionnaire, developed using items that represent the indicators, was conducted to generate categories of responses. The results of the questionnaire reflected the need for an in-depth exploration of the indicators. Focus groups were conducted prior to modifying the indicators and categories. They are currently being used to develop the research questionnaire. The results will be used to suggest recommendations for facilitating student learning, improving teacher preparation programs, and modifying the mathematics curricula to prepare students for the challenges of the global economy.

**Keywords:** conceptions, dispositions, mathematics teaching, mathematics learning, indicators

## INTRODUCTION

Preparing students to live and work in a rapidly changing technological world poses serious challenges to educational systems. The demands for quantitative literacy are changing, with the modern workforce needing critical thinking, problem solving and data analysis skills in addition to the basic mathematical skills. Along with the cognitive aspects of teaching and learning mathematics, research in mathematics education is focusing on affective variables to come to a better understanding of individual learners and how they interact with their learning environment. Interest in affective variables is attributed to their impact on how students learn and use mathematics, as well as the potential of these variables to hinder effective learning (Gomez-Chacon, 2001).

Mathematics has a public image of being a difficult subject, accessible only to the few. Learners who do well in mathematics are typically stereotyped as “nerds”. Mathematics is generally disliked. It is seen as a dry and boring subject. Often, it evokes feelings of stress; anxiety and fear (see Zaslavsky, 1994). Furthermore, it is seen as a filter that hinders students from pursuing their career aspirations (see Ernest, 1994; National Research Council, 1989).

This research builds on a study on student conceptions of mathematics, conducted in the United Arab Emirates (UAE) (see Atallah 2003, 2004). The researchers’ interest in investigating conceptions, and extending their study to dispositions of mathematics, stems from their classroom observations as mathematics educators, as well as from the mathematics education research. Student conceptions of mathematics affect the quality of their learning (Frid and White, 1995). These conceptions are influenced by the students’ classroom experiences (Schoenfeld, 1989). In addition, student dispositions towards mathematics are one of several factors that affect student learning (NCTM, 1989). Thus, the way students see mathematics and what they believe it to be affects the way they approach it as a subject and how they react in the mathematics classroom. Furthermore, teacher conceptions of mathematics influence their teaching approaches (Kuhs and Ball, 1986). Teacher conceptions are influenced by their own dispositions towards the subject being taught (Damon, 2005).

## STATEMENT OF THE PROBLEM

This paper discusses an on-going research study that investigates student and teacher conceptions and dispositions of mathematics. Conceptions of mathematics, as adapted from Oaks’s (1994) definition, refer to views that students hold of a subject, and what they believe is required in learning and doing it. Dispositions of mathematics, as adapted from Katz’s (1993) definition, refer to beliefs or tendencies to exhibit a frequent, conscious and voluntary behavior directed towards learning a subject.

Conceptions and dispositions exist in a symbiotic relationship. While seeking to explore student and teacher conceptions and dispositions of mathematics from a UAE perspective, the researchers believe that these themes are global, and that mathematics educators in different parts of the world share related concerns. These conceptions and dispositions influence student learning in a mathematics classroom. Thus the researchers’ interest in an international dialogue to share experiences, insights and strategies to come to a better understanding of the relation between student and teacher conceptions and dispositions of mathematics.

The study addresses the following questions: (1) What are student/teacher conceptions of mathematics as a discipline? (2) What are student/teacher conceptions of the application of

mathematics in everyday life? (3) What are student/teacher conceptions of the links between mathematics and its applications in the work environment? (4) How do student/teacher conceptions towards mathematics influence their dispositions of how to learn/teach the subject? (see Atallah, Bryant & Dada, 2009).

### **Overview of Conceptions and Dispositions**

Thompson (1992) refers to conceptions as mental structures that encompass beliefs, concepts, meanings, propositions, mental images and other. She suggest that the distinction between conceptions and beliefs is not “a terribly important one” when talking about teacher conceptions of mathematics as a discipline and teacher beliefs about mathematics (Thompson, 1992, p. 130). Studies on teacher conceptions and beliefs have focused on describing teacher beliefs and conceptions, on examining the relation between teacher conceptions and instructional practices, or on changing teacher conceptions of mathematics (Thompson, 1992). Andrews and Hatch (2000) suggest that the literature on conceptions is not clear because different researchers offer different perspectives on conceptions in terms of having cognitive and/or affective dimensions. Oaks (1994) describes conceptions as views that students hold of mathematics and what they believe is required in learning and doing mathematics.

Damon (2005) describes dispositions as traits or characters that lead a person to follow certain choices or experiences. Damon views dispositions as having a major impact on who we are and who we become. According to the National Council of Teachers of Mathematics [NCTM] (1989, p. 233), a mathematical disposition refers to "a tendency to think and act in positive ways". Katz (1993) defined the term dispositions as a “tendency to exhibit frequently, consciously and voluntarily a pattern of behavior that is directed to a broad goal”. Rath (2001) views dispositions are closely related to skills and practices.

### **RESEARCH FRAMEWORK**

The research framework that emerged from the literature review, as well as from the researchers’ perspectives, included the identification of six indicators for each of the conceptions and dispositions. Each of the two sets of indicators form a circle with six equal parts. The research framework diagram depicts two non intersecting circles, one representing conceptions and one disposition (see Diagram 1). The indicators have been modified based on the results of the first stage of the study, where an open ended questionnaire and focus groups were conducted with undergraduate students at a university in the Arabian Gulf region. The exploratory data collected and analyzed to-date suggests a dynamic interrelationship between conceptions and dispositions as well as sections within each of conceptions and dispositions. Further analysis will continue to shed light on the interactions among and between the two.

The six modified indicators of conceptions of mathematics are:

- C1 Describing what mathematics is – ideas or thoughts about the nature and origin of mathematics (science) e.g. invented or created), (absolute or relative), (static or dynamic), (numbers and rules, patterns, logical process ...)
- C2 Drawing what mathematics is – one’s mental image of the subject
- C3 Describing what is required to learn mathematics outside the class
- C4 Describing what in-class activities help one learn mathematics
- C5 Describing the purpose of learning mathematics

C6 Describing when thoughts about the mathematics concept under study come together i.e. when is it understood

The six modified indicators of dispositions of mathematics:

D1 Describing one's own ability in mathematics as a learner

D2 Describing one's attitudes towards mathematics i.e. feelings, emotions, interests

D3 Describing what will mathematics helps one achieve

D4 Describing the learning approach used to study mathematics

D5 Describe the perceived value of mathematics i.e. the public image

D6 Describing the evidence that proves one's learning of mathematics

### **Conceptions Indicators**

A review of the literature presents several views on describing student and teacher conceptions of mathematics. Some research on conceptions of mathematics focuses on the nature and functions of mathematics as well as views on how one learns mathematics. In their study on teacher and student conceptions, Frid and White (1995) and White and Frid (1995) include questions related to the origins of mathematics and whether it was discovered or created. In their study on student conceptions, Grouws et al. (1996) includes items related to the nature, structure and status of mathematics and mathematical knowledge. Gibson (1994) uses a writing exercise to examine student conceptions of mathematics. Amongst other things, Gibson asks students to list words or phrases to describe mathematics, and to complete a sentence that is a metaphor for what mathematics is most like. In this study, the researchers identified the indicators C1 and C2 to addresses student views on the nature of mathematics. While the C1 indicator is reflected in writing, the C2 indicator is reflected in drawing. Some are better able to express their thoughts in drawing as they may have more visual learning styles. Furthermore, imagery reveals underlying beliefs, assumptions and expectations (Henrion in Picker and Berri, 2000).

Frid and White (1995) and White and Frid (1995) also included questions related to factors that facilitate the learning of mathematics, and specific mathematics teaching and learning experiences. Grouws et al. (1996) included items on the nature of mathematical activities. In this study, the researchers identified the indicators C3, C4 and C6 to reflect the relation between learning and doing in mathematics where knowledge and skills are complimentary.

Frid and White (1995) and White and Frid (1995) also addressed questions related to the purpose of studying mathematics and why it is included in school programs. Grouws et al (1996) used items related to the usefulness of mathematics. In this study, the researchers included the C5 indicator to reflect the purpose of studying mathematics.

### **Dispositions Indicators**

Perkins and Tishman (1998) use the term disposition to refer to a predilection to exhibit a behavior under certain conditions. They suggest that dispositions involve sensitivity, inclination and ability. Sensitivity concerns awareness to the environment. Inclination concerns motivation or learning. Ability concerns capability to follow through appropriately. In this research, the inclusion of indicator D1 reflects the ability to learn. The inclusion of indicators D4 and D6 relates to the behavioral aspect of a disposition included in the definition.

In a study on fostering student dispositions, Anku (1996) uses change in attitudes before and after the experimental treatment as an indicator of the disposition. Bonnsteter's (2003) research uses three types of instruments for measuring teacher dispositions – one that measure observable behavior and emotions, one for attitudes and values, and one for soft skills. Osborne et al (2003) discuss attitudes towards a subject (in this case science) as encompassing feelings, beliefs and values related to the subject. NCATE defines dispositions, amongst others, as values, that influence behavior and affect student learning. Howes (2002) defines teacher dispositions towards science as outlooks, attitudes, and expectations concerning one's relationship with science, as well as toward learners. In this study, the indicator D2 was included to address attitudes related to mathematics. The indicator D5 was included to address perceived values of mathematics, and the indicator D3 was included to address expectations regarding mathematics.

### **Conceptions and Dispositions**

The authors suggest that the six indicators of disposition represent the pre-conditions necessary for the interaction between dispositions and their conceptions of mathematics.

## **METHODOLOGY**

### **Participants**

The participants at the exploratory stage of the research study were undergraduate students from different areas of specialization at a public university in the United Arab Emirates. At the next stage, the participants will include high school students, undergraduate students from other public institutes of higher learning as well as school, college and university mathematics instructors.

### **Procedures**

The study utilizes qualitative and quantitative procedures, with triangulation of data through using questionnaires, interviews and focus groups. The study also emphasizes the development of reliable and valid instruments to measure student and teacher conceptions and dispositions of mathematics. The research process is iterative in nature and includes a review of literature, designing the exploratory and study questionnaires, data collection and analysis, results and recommendations, research disseminating and cross-cultural extension. Currently, the researchers are in the process of designing the research instruments using the results of the exploratory questionnaire. This process feeds into modifying the research framework and generating categories for the conceptions and dispositions indicators. These categories are used to construct the items included in the study questionnaire. This questionnaire will be piloted prior to conducting it on-line.

## **PRELIMINARY RESULTS AND DISCUSSION**

Results from the data analysis of the exploratory questionnaire reflect the need for a more in-depth examination of some indicators and their corresponding questionnaire items. Furthermore, the categories generated for some items appear too dispersed. Some items required

further investigation such as the one on problem solving processes. Even with the focus group approach, it was still evident that students have misconceptions about the nature of problem solving and what it involves in mathematical situations. For the most part, these students equate problem solving to finding the correct answer to an exercise or exam question.

Two focus group sessions were carried out following the exploratory questionnaire. An interesting observation stemmed from comparing answers of both focus groups. The differences between the group enrolled in a third year Learning Technologies course, which comprised students from different education majors (Early Childhood Studies, Primary Mathematics and Sciences Studies, Mathematics Studies, Social Studies, English Studies, Art Studies, and Technology Education Majors) and the group enrolled in a second/third year Mathematics Teaching Methods II course, which included both Primary Mathematics and Sciences Studies and Mathematics Studies, seem to suggest a connection with the major course of study. For example, in the Learning Technologies group, some answers for an item on studying reflected a reliance on language strategies for learning terms rather than concepts (syllabus, stories,). This issue will be further explored in the main study.

The researchers have used the data analysis results to review and modify the indicators as well as the categories. We are in the process of constructing and piloting the study questionnaire. This questionnaire will be followed up with other focus groups and interviews. While providing valuable data, focus groups do not always provide the in-depth perspectives needed on truly understanding conceptions and dispositions towards learning math. Interviews may shed further light on understanding these complex and sometimes interrelated issues. Because of the complex nature of the human mind where conceptions and dispositions lie, any type of investigation must analyze the interrelations between conceptions and dispositions, and the connections within and between them.

## FUTURE PERSPECTIVES

The study results will be used to suggest recommendations for facilitating student learning, improving teacher preparation programs, and modifying the mathematics curricula to become more relevant to the challenges of the global economy. As concerns relating to mathematics education are shared by mathematics educators across the world, the authors are interested in cross cultural studies that explore conceptions and dispositions of mathematics as well as the relationship between them, and its implications for the teaching and learning of mathematics.

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Diagram 1: Research Framework on Mathematics Conceptions and Dispositions

