Corporate cost of debt: the issue of premium or discount bonds

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ABSTRACT

The traditional textbook method of calculating a corporation's cost of debt capital tends to minimize the practical process used to arrive at that cost. This is particularly true if the corporation has issued a coupon bond at either premium or discount. The theoretical before-tax cost of debt is easily calculated and does include the amortization of the discount or premium, but the significance of this amortization is not apparent. Further undermining the cost-of-capital process, finance and accounting students are taught that the dollar amount of coupon interest is tax-deductible to the corporation while tending to ignore the tax effect of amortizing the discount or premium or the amortization of other expenses associated with the original issuance of the bond. This paper bridges finance theory with accounting practice to fully include the effect of discount or premium amortization on the cost of debt capital to a corporation and extends material presented in popular finance and accounting textbooks.

Keywords: Cost of debt, bonds, capital, amortization, premium, discount

I. INTRODUCTION

As typically introduced to students of finance, the corporate cost of debt is a simple matter of determining the yield to maturity that investors demand from the corporation's outstanding bonds (Besley & Brigham, 2004, pg. 339-340; Block & Hirt, 2005, pgs. 278 and 314, Brigham and Daves 2004, pgs. 299 and 318). The yield is then adjusted for the corporation's marginal tax rate. As a practical matter, the actual cost of debt also depends on several other factors which include:

1. Floatation costs that are paid by the corporation when the debt is issued (consisting of the underwriting spread, legal, regulatory, and other expenses),
2. The relationship between the yield to maturity demanded by new bondholders and the coupon rate of outstanding debt,
3. The timing of the coupon payments,
4. The corporation's marginal tax rate after debt is issued,
5. Incentives or features that the bonds may carry (e.g. convertible, callable, sinking fund, warrants, etc.), and
6. The choice of accounting method used to record the coupon interest, the amortization of a premium or discount, and the expenses related to the bond issuance.

The traditional bond pricing formula consists of discounting the future coupon payments and the lump sum maturity value to arrive at a fair market price for the bond. If the market price of the bond is known, the yield to maturity, r_d, may be determined. The yield to maturity is assumed to represent the before-tax percentage cost of debt to the corporation. This form is depicted by equation (1):

$$(1) \qquad FMP_0 = \sum_{t=1}^{N} \frac{CI}{[1+rd]^t} + \frac{PAR}{[1+rd]^N}$$

Where:

FMP_0	=	Current fair market price of the bond if r_d is known.
PAR	=	Maturity or Par value to be returned to investor upon maturity.
N	=	Total number of periods until the bond matures.
t	=	Individual time periods until maturity.
CI	=	Dollar amount of interest paid per coupon period.
r_d	=	The periodic before-tax cost of debt stated as a percentage (*also the yield to maturity*), if the current market price of the bond is known.

Intermediate finance texts attempt to include the cost to the corporation associated with debt while simultaneously including other factors such as floatation costs and changing corporate tax rates. Brigham and Daves (2004, pgs. 318-319) claim the after-tax cost of debt, r_d, may be found with their equation (9-8):

$$(2) \qquad PAR(1-F) = \sum_{t=1}^{N} \frac{CI(1-T)}{\left[1+rd(1-T)\right]^t} + \frac{PAR}{\left[1+rd(1-T)\right]^N}$$

Where:

F	=	Percent of par value paid in floatation costs.
T	=	Corporation's marginal tax rate.
r_d	=	*(incorrectly identified as)* The periodic after-tax cost of debt adjusted for floatation costs. *(actually the before-tax cost of debt adjusted for floatation costs).*

The text provides an example using Equation (2); however, the example assumes the bond was originally issued at par so the before-tax yield to maturity is equivalent to the coupon rate offered on the bond [the reason PAR(1-F) appears as the dependent variable]. This assumption is consistent with the presentation but it ignores the effect on corporate cost of debt if the bond were issued at a premium or a discount.

Later in the text, Brigham and Daves (2004, pgs. 617-620) address the effect on the cost of debt capital if zero coupon (or very low coupon) bonds are issued. An example is presented and generalized to include original issue discount bonds with coupon payments. However, the amortization of original issue *premium* bonds and the effect on the after-tax cost of debt is not explored.

Larson, Wild and Chiappetta (2005, pgs. 551-560) approach the corporate cost of debt from a practical accounting standpoint. The text provides examples of the dollar interest expense that is recorded by a corporation after issuing either a premium or a discount bond series. Further, floatation costs are assumed to be included as part of interest expense. This approach recognizes the need to include tax effects of the floatation costs, but a comprehensive theoretical form for the expenses and the amortization of the discount or premium is not provided.

Often unexplored in finance and accounting texts is the effect on the cost of debt of issuing bonds with coupon rates that are not equal to the yield demanded by investors. Most students of finance understand that this condition causes the individual bonds to initially sell at a premium or a discount from par value. In the case of a discount bond, the coupon rate is insufficient to compensate investors, so additional compensation is sought by bidding less than the par value. The opposite occurs for premium bonds. The coupon rate is desirable because it is above investors' required yield and the price is bid up with the understanding that the premium over par value will not be returned.

Complicating the issue is that fact that more than one type of floatation cost is faced by a corporation. First, an underwriting spread, deducted by investment bankers, is a percentage of the market price paid by investors for the bonds. Presumably, this spread can deepen the effective discount on a bond or reduce the premium that is received by the corporation. Second, other types of expenses associated with the issuance of bonds, such as legal and printing expenses, should also be amortized over the life of the bonds under U.S. GAAP (Kieso, Wygandt, and Warfield, 2001, pgs. 722-723)

This paper addresses the corporate cost of debt when floatation costs as well as the amortization of a discount or premium coupon bond are considered. It closes a theoretical gap occasionally mentioned in popular finance texts but not fully explored. The paper recognizes that intermediate level accounting texts often include actual market practice as outlined by the Internal Revenue Service, but the texts lack a theoretical finance base.

Focusing on the cost of debt capital, the paper creates a bridge between introductory finance, intermediate finance, accounting, and corporate practice. The traditional bond pricing formula is used as a starting point and new terms are introduced to incorporate and provide the theoretical finance base underlying actual corporate practice. Differences between this new approach and the traditional theoretical approach are then compared. Finally, it is shown that a preferential accounting method does exist for recording the expenses, interest, and amortization of discounts or premiums of a new bond issue.

II. THE BEFORE AND AFTER-TAX COST OF CORPORATE DEBT: TRADITIONAL FINANCE APPROACH

Traditionally, the before-tax cost of debt (r_d) is equivalent to the yield to maturity when the current market or issue price of a coupon bond is (MP_0) is known:

(1) $$FMP_0 = \sum_{t=1}^{N} \frac{CI}{\left[1+rd\right]^t} + \frac{PAR}{\left[1+rd\right]^N}$$

Since this method combines discounting an ordinary annuity as well as a lump sum maturity value, there is no simple closed-form solution. An iterative process must be used to determine the appropriate yield. The introduction of financial calculators has greatly reduced the effort required to perform this process. Once the before-tax cost of debt has been determined, the traditional method uses a simple adjustment for marginal tax rates to arrive at an after-tax cost of debt (r_{dT}):

(3) $$r_{dT} = r_d\left(1-T\right)$$

<u>EXAMPLE 1: Discount bond and traditional finance practice</u>

According to finance theory, a corporate coupon bond issued with the following characteristics will provide an investor an annual yield to maturity of 8.2619%.

a. Maturity value of $1,000.
b. Coupon rate of 6%.
c. Annual interest payments.
d. Term to maturity of 10 years.
e. Corporation pays taxes at the marginal rate of 40%.
f. The corporation receives $850 after floatation costs for each bond sold.

From the corporation’s perspective, this bond carries a before-tax cost of 8.2619%. The traditional approach would then adjust for the 40% marginal tax rate of the corporation so that the after-tax cost of debt is determined to be: 8.2619(1-0.40) = 4.9572%. This approach applies the tax adjustment to all components of the return such as outright coupon interest expense and amortizable expenses such as the discount and underwriting spread because the $850 received by the corporation is net of these costs.

<u>EXAMPLE 2: Discount bond and the Brigham and Daves method</u>

According to Brigham and Daves (2004, pgs. 299 and 318) the after-tax cost of debt can be found directly if one knows the marginal tax rate in addition to the amount received by the corporation on each bond. Making use of equation (2) and the assumptions of Example 1, the solution for r_d is 5.5995%. Note that a financial calculator will easily determine that r_d(1-T) is equal to 9.33255%, or the unadjusted before-tax cost.

$$850 = \sum_{t=1}^{10} \frac{60(1-0.40)}{\left[1+r_d(1-0.40)\right]^t} + \frac{1{,}000}{\left[1+r_d(1-0.40)\right]^{10}} = \sum_{t=1}^{10} \frac{36}{\left[1+r_d(0.60)\right]^t} + \frac{1{,}000}{\left[1+r_d(0.60)\right]^{10}}$$

This bond is initially sold at a discount; because the coupon of 6% is insufficient to compensate investors who *apparently* demand a return of 9.33255%. The form appears to be consistent with the after-tax cost of debt as traditionally taught because the after-tax coupon payment is incorporated in the solution. However, the actual amount that the corporation receives is $150 below the amount to be paid at maturity. Therefore, the corporation will not only pay periodic coupon payments, but also $150 more at maturity than initially received. In effect, this amount represents additional return to the investor and additional cost to the corporation. The Brigham and Daves method does not specifically take this extra cost into account because their form does not include any adjustment for the amortization of a premium or discount. Therefore, both the before and after-tax cost of debt is higher than the traditional method for a bond originally issued at a discount.

In practice the percent of par value paid in flotation costs ["F" in equation (2)] should include the discount or premium, along with "engraving and printing costs, legal and accounting fees, commissions, promotion costs and other similar charges." (Kieso, Wygandt, and Warfield, 2001, p. 722). Since all costs of issuance should be amortized over the life of the bond, their incorporation into the derivation in this fashion is appropriate, although not identified as such. However, the maturity portion of the bond price formula should also contain an adjustment for the premium or discount and the resultant tax effect.

III. THE BEFORE AND AFTER-TAX COST OF CORPORATE DEBT: TRADITIONAL ACCOUNTING APPROACH

Finance academics emphasize that cash flows are the primary focus of virtually all transactions that occur. Finance practitioners communicate returns and costs of capital stated as percentages. The field of accounting is routinely concerned with dollar amounts because of a primary emphasis on the integrity of an accounting system. In the case of corporate capital raised through the issuance of bonds, the communication of the costs involved requires a careful consideration of both the actual dollars received, the cash flows involved, and the percentage cost that are incurred.

Two methods exist that may be used to report the actual interest expense recorded for bonds that are issued at a premium or discount. Larson, et.al. (2004) explain both the straight-line method and the effective interest method. Each method carries different implications in terms of correctly identifying the cost of debt capital, since the time distribution of the dollar amounts to be amortized will affect the present value of the difference in deductible expenses. The time distribution of these expenses is critical to practitioners and academics in the field of finance to arrive at an appropriate percentage cost of capital.

a. The straight-line method of bond amortization

The straight-line method of amortizing the expenses of issuing the bond and the discount or premium assumes:

1. Total floatation costs are deducted in equal dollar amounts each coupon period over the life of the security.
2. If the funds received net of floatation costs are greater than the par value, the difference is deducted in equal dollar amounts each coupon payment period until maturity.
3. If the funds received net of flotation costs are less than the par value, the difference is added in equal dollar amounts each coupon payment period until maturity.

Using these assumptions, the traditional bond price formula can be represented by Equation (4):

$$(4) \qquad AR_0 = \left[\sum_{t=1}^{N} \frac{CI}{(1+r_d)^t} + \frac{\frac{(PAR - AR_0)}{N}}{(1+r_d)^t} \right] + \left[\frac{PAR}{(1+r_d)^N} \right]$$

Where:

AR_0 = Amount received by the corporation *after all floatation costs*.

The introduction of a new term, which represents the constant dollar amount added to interest expense each coupon period, reduces Equation (4) to:

$$(5) \qquad AR_0 = \left[\sum_{t=1}^{N} \frac{(CI + \Delta)}{(1+r_d)^t} \right] + \left[\frac{PAR}{(1+r_d)^N} \right]$$

Where: $\Delta = \left[\frac{PAR - AR_0}{N} \right]$

The form of Equation (5) encompasses both discount and premium bonds. For premium bonds, Δ is negative as the amount received when the bonds are issued is larger than the commitment at maturity. Therefore, the cash flow recorded by the corporation for interest expense will be *lower* than the actual cash flow paid to bondholders. The difference takes into account the amortization of the premium associated with the original issue.

In the case of bonds issued at a discount, interest expense recorded will be *higher* than the actual coupon payment to bondholders. The amortization of the bond as it moves toward maturity is recorded as additional interest expense each coupon period although the cash flow at maturity has yet to occur.

<u>EXAMPLE 3: Effect of straight-line method on the corporate cost of debt</u>

Assume the same characteristics as Example 1 and further assume that a total of 1,000 bonds are issued so the corporation receives $850,000 after all costs related to the issue are incorporated. Under the straight-line method of accounting for the $150,000 discount from par value, the discount is then equally distributed over the 10 payment periods and added to the $60,000 recorded as the coupon payment to bondholders. The corporation will record $75,000 as interest expense each coupon payment period.

The field of finance, however, focuses on the percentage cost of capital to the corporation. When a corporation chooses to use the straight-line accounting method, the percentage cost of debt capital changes each period because the book value of the bond issuance changes while the interest expense recorded remains constant. The effect of this choice is rarely explored in the academic fields of either accounting or finance. Table 1 contains a predetermined schedule of interest expense and before and after-tax cost of capital based on the example.

b. The effective interest method of bond amortization

The effective interest method of accounting for bond amortization is more consistent with finance theory than the straight-line method of accounting for interest expense and bond discount or premium amortization. This method assumes:

1. Floatation costs are deductible at each coupon period over the life of the security, and
2. In addition to each coupon payment, the corporation records as interest expense the change in the amortized value of the bond each coupon period. Since the amortized value of the bond does change over time, the total dollar amount of interest expense for any given period will change as well.

This method allocates the amortization of a discount or premium in such a way that the before-tax bond yield (cost) is a constant *rate of interest*, rather than a *constant dollar amount*, as in the straight-line method. That is, the result is a constant rate of geometric average growth.

In order to capture the change in the non-constant amortized value of the bond each coupon period, a new term is introduced to the traditional bond pricing formula. Equation (6) is a representation of the underlying theoretical form used by a corporation that has adopted the effective interest method to record periodic interest expense which includes the amortization of a discount or premium and floatation costs. A corporation's cash outflow of coupon interest is obviously tax deductible. The inclusion of a term for the change in amortized value of the security is not usually obvious in finance theory, nor is its effect on the corporate cost of debt. As a practical accounting matter, however, the amortized dollar value of a discount or premium affects the actual interest expense recorded and reported in financial and tax documents.

$$AR_0 = \left[\sum_{t=1}^{N} \frac{CI}{(1+r_d)^t} + \sum_{t=1}^{N} \frac{\left(AV_{N-t} - AV_{N-(t-1)}\right)}{(1+r_d)^t}\right] + \left[\frac{PAR}{(1+r_d)^N}\right] \qquad (6)$$

Where:

AV_{N-t} = Non-constant amortized value of the bond.

The second term in Equation (6) depicts the periodic change in the amortized value of the bond that is included with the coupon payment as interest expense on the corporation's financial statements that chooses this method of accounting. Finance theory indicates that this change in value may be written as:

(7)

$$AV_{N-t} - AV_{N-(t-1)} = \left[CI\left(\frac{1}{r_d} - \frac{1}{r_d\left(1+r_d\right)^{N-t}} \right) + \frac{PAR}{\left(1+r_d\right)^{N-t}} \right] - \left[CI\left(\frac{1}{r_d} - \frac{1}{r_d\left(1+r_d\right)^{N-(t-1)}} \right) + \frac{PAR}{\left(1+r_d\right)^{N-(t-1)}} \right]$$

Appendix I contains the complete derivation that reduces the difference between the amortized value of a bond each coupon period represented by equation (7) to the following:

(8) $$AV_{N-t} - AV_{N-(t-1)} = \frac{r_d PAR - CI}{\left(1+r_d\right)^{N-(t-1)}}$$

The entire cost of debt capital equation, including amortization of the bond premium or discount and floatation costs can then be represented by equation (9):

(9) $$AR_0 = \left[\sum_{t=1}^{N} \frac{\left(CI + \left[\dfrac{r_d PAR - CI}{\left(1+r_d\right)^{N-(t-1)}} \right] \right)}{\left(1+r_d\right)^{t}} \right] + \left[\frac{PAR}{\left(1+r_d\right)^{N}} \right]$$

The inclusion of the unknown before-tax cost of debt, r_d, in both the numerator and denominator of the first term eliminates the possibility of a true closed-form solution, as with the bond valuation formula used to find the yield to maturity. Either form requires an iterative technique to find the implied percentage cost of debt capital.

<u>EXAMPLE 4: Effect of the effective interest method on the corporate cost of debt</u>

Using the same assumptions as Example 3, a corporation that chooses the effective interest method to account for floatation costs and the amortization of the bond discount will record an interest expense each coupon period as detailed in Table 2. As noted, this dollar amount changes each period because of the changing amortization of the original discount. The fifth and sixths columns of Table 2 contain the before and after-tax cost of debt, respectively. In keeping with traditional finance theory, this percentage cost does not change during the life of the bond issue and it is equivalent to the traditional method taught to students of finance.

The average cost of debt from the straight-line method of accounting for floatation costs and amortization results in a lower estimate of the cost of debt for an original issue discount bond than the effective interest method. If the bond were originally issued at a premium, the strict average cost of debt from the straight-line method would be slightly higher than the effective interest method.

IV. TIME VALUE OF MONEY CONSIDERATIONS WHEN CHOOSING A METHOD FOR FLOATATION COSTS AND AMORTIZATION OF A DISCOUNT OR PREMIUM OF A CORPORATE BOND ISSUE

a. Discount bond issues, when all costs are considered

Due to the difference in the actual dollar interest expense recorded between the straight-line and the effective interest methods, it is relevant in finance theory to consider which choice may benefit the corporation over time. Table 4 displays the interest expense recorded under each method and established in Example 3 (straight-line method) and Example 4 (effective interest method) when the issue is originally sold at a discount.

If the straight-line method is chosen, the difference in actual tax-deductible interest expense by the corporation that has issued a discount bond is positive in the early years and negative in later years. Finance theory pertaining to the timing of cash flows suggests that this is a desirable situation. The traditionally determined before-tax cost of debt (8.2619 %) is used as a discount rate for the difference in these cash flows. The positive result of $5,013.59 leads to the conclusion that a corporation which nets less than par value from an issue of bonds after all costs and yields are considered would be wise to choose the straight-line method of accounting for coupon interest, amortizable expenses, and the bond discount. In doing so, the corporation receives a net benefit over the alternative accounting method.

b. Premium bond issues, when all costs are considered

It is instructive to consider the effect of an accounting choice when a corporation receives an amount greater than par, after all costs and yields are considered, under similar circumstances presented throughout this paper. As is often the case, the theoretical result for a premium bond issue is opposite that of a discount bond issue.

Assume the following for a corporate coupon bond issue:

a. Maturity value of $1,000.
b. Coupon rate of 10%.
c. Annual interest payments.
d. Term to maturity of 10 years.
e. Corporate marginal tax rate of 40%
f. Corporation receives $1,200 after all floatation and regulatory costs for each bond sold.
g. A total of 1,000 bonds are issued.

Under this scenario, the corporation receives an amount greater than the obligation due at maturity because the coupon rate is such that it outweighs floatation and other costs of the bond issue. It can be determined that the before-tax percentage cost to the corporation is 7.1347% after all costs and yields are considered.

Column 1 of Table 4 displays the amounts recorded by a corporation that chooses the straight-line method to account for the premium over time. The $200,000 premium is equally divided over 10 coupon periods, and the result is subtracted from the actual coupon payment to bondholders of $100,000 per year. The interest expense recorded each coupon period is $80,000.

Under the effective interest method, the constant before-tax percentage cost of capital is found to be 7.1347%, which is aligned with traditional finance theory. The interest expense recorded by a corporation using the effective interest method is contained in Column 2 of Table 4, and results in a non-constant schedule of recorded interest expense.

The corporation that chooses to amortize the premium of $200,000 using the straight-line method will record differentially lower interest expense in the early years of the bond issue and higher interest expense in later years than if the effective interest method were chosen.

Finance theory suggests that this is a *not* a desirable situation. The net present value of the future cash flows is negative using a discount rate equal to the before-tax cost of debt capital under this circumstance. The negative result of $5,343.40, leads to the conclusion that if a corporation nets more than par value from an issue of bonds, after all costs and yields are considered, it is wise to choose the effective interest method of accounting for coupon interest, amortizable expenses, and the bond premium. The corporation will receive a net benefit from this choice of accounting methods.

V. CONCLUSION

It is seldom apparent to students of finance and students of accounting the intricacies involved in determining the true cost of debt capital to a corporation. When coupon bonds are originally issued, the corporation must consider legal, regulatory, floatation, and other costs when preparing future financial documents. In addition, the bond issue may initially sell at a price above or below par value. All of these factors have an effect on the true cost of debt capital to the corporation.

When coupon bonds are issued, the corporation has a choice of methods to account for the flotation costs and amortization of a premium or discount from par value. The straight-line method simply averages the costs and difference from par value over the life of the bond and deducts, or adds in the case of a premium, the same amount to actual coupon interest expense each period. This paper presents a modified and extended theoretical bond valuation formula that incorporates this relatively simple adjustment.

Similarly, the paper also presents, derives, and simplifies a more complicated adjustment factor to the traditional bond valuation formula when the effective interest method is chosen to account for floatation costs and amortization of a discount or premium. A corporation that chooses this method will find itself aligned with traditional finance theory when determining the percentage cost of debt capital.

Finally, the time value of money effect of the two choices of accounting methods is detailed. The examples provided show that if a corporation receives less than par value after all costs and yields are considered, it is wise to choose the straight-line method of accounting for these factors when preparing financial statements. The choice of the straight-line method will provide more benefit than the alternative. If a corporation receives an amount greater than par value after all costs and yields are considered, it is best to choose the effective interest method to account for the amortization of the costs and bond premium. In following this strategy, the corporation will realize a positive net present value simply by making the correct choice of accounting methods.

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APPENDIX I – Formulas

Derivation of Solution to Effective Interest Method of the Amortization of the Discount or Premium of a Coupon Bond

The same variable definitions apply as used in the text of the paper.

(6) $$AR_0 = \left[\sum_{t=1}^{N} \frac{CI}{[1+rd]^t} + \sum_{t=1}^{N} \frac{\left(AV_{N-t} - AV_{N-(t-1)}\right)}{[1+r_d]^t} \right] + \left[\frac{PAR}{[1+rd]^N} \right]$$

The terms $AV_{N-t} - AV_{N-(t-1)}$ represent the change in the amortized value of the bond each coupon period. Making use of the present value of an annuity factor formula, the change in amortized value is mathematically equivalent to:

(7) $$\left[CI\left(\frac{1}{r_d} - \frac{1}{r_d\,(1+r_d)^{N-t}} \right) + \frac{PAR}{(1+r_d)^{N-t}} \right] - \left[CI\left(\frac{1}{r_d} - \frac{1}{r_d\,(1+r_d)^{N-(t-1)}} \right) + \frac{PAR}{(1+r_d)^{N-(t-1)}} \right]$$

And factoring like terms:

$$AV_{N-t} - AV_{N-(t-1)} = \left[\frac{CI}{r_d}\left(1 - \frac{1}{(1+r_d)^{N-t}} \right) + \frac{PAR}{(1+r_d)^{N-t}} \right] - \left[\frac{CI}{r_d}\left(1 - \frac{1}{(1+r_d)^{N-(t-1)}} \right) + \frac{PAR}{(1+r_d)^{N-(t-1)}} \right]$$

Collecting terms

$$AV_{N-t} - AV_{N-(t-1)} = \left[\frac{CI}{r_d}\left(1 - \frac{1}{(1+r_d)^{N-t}} \right) - \frac{CI}{r_d}\left(1 - \frac{1}{(1+r_d)^{N-(t-1)}} \right) \right] + \left[\frac{PAR}{(1+r_d)^{N-t}} - \frac{PAR}{(1+r_d)^{N-(t-1)}} \right]$$

$$AV_{N-t} - AV_{N-(t-1)} = \frac{CI}{r_d}\left(1 - \frac{1}{(1+r_d)^{N-t}} - 1 + \frac{1}{(1+r_d)^{N-(t-1)}} \right) + PAR\left(\frac{1}{(1+r_d)^{N-t}} - \frac{1}{(1+r_d)^{N-(t-1)}} \right)$$

Reducing and rearranging:

$$AV_{N-t} - AV_{N-(t-1)} = \frac{CI}{r_d}\left(\frac{1}{(1+r_d)^{N-(t-1)}} - \frac{1}{(1+r_d)^{N-t}} \right) + PAR\left(\frac{1}{(1+r_d)^{N-t}} - \frac{1}{(1+r_d)^{N-(t-1)}} \right)$$

The next step is crucial as the substitution of $(1+r_d)/(1+r_d)$ into certain terms does not change the result. The value of one is added to the exponent of one of the denominators. This substitution allows a substantial reduction of the overall formula.

$$AV_{N-t} - AV_{N-(t-1)} = \frac{CI}{r_d}\left(\frac{1}{(1+r_d)^{N-(t-1)}} - \frac{\mathbf{(1+r_d)}}{(1+r_d)^{\mathbf{N-(t-1)}}}\right) + PAR\left(\frac{\mathbf{(1+r_d)}}{(1+r_d)^{\mathbf{N-(t-1)}}} - \frac{1}{(1+r_d)^{N-(t-1)}}\right)$$

Simplifying:

$$AV_{N-t} - AV_{N-(t-1)} = \frac{CI}{r_d(1+r_d)^{N-(t-1)}}(1-1-r_d) + \frac{PAR}{(1+r_d)^{N-(t-1)}}(1+r_d-1)$$

$$AV_{N-t} - AV_{N-(t-1)} = \frac{-r_d CI}{r_d(1+r_d)^{N-(t-1)}} + \frac{r_d PAR}{(1+r_d)^{N-(t-1)}}$$

$$AV_{N-t} - AV_{N-(t-1)} = \frac{r_d PAR}{(1+r_d)^{N-(t-1)}} - \frac{CI}{(1+r_d)^{N-(t-1)}} = \frac{r_d PAR - CI}{(1+r_d)^{N-(t-1)}}$$

(8) $$AV_{N-t} - AV_{N-(t-1)} = \frac{r_d PAR - CI}{(1+r_d)^{N-(t-1)}}$$

Substituting into the original equation (6) results in:

$$AR_0 = \left[\sum_{t=1}^{N}\frac{CI}{[1+rd]^t} + \sum_{t=1}^{N}\frac{\left(\frac{r_d PAR - CI}{(1+r_d)^{N-(t-1)}}\right)}{[1+r_d]^t}\right] + \left[\frac{PAR}{[1+rd]^N}\right]$$

Reducing results in equation (9):

(9) $$AR_0 = \left[\sum_{t=1}^{N} \frac{\left(CI + \left[\frac{r_d PAR - CI}{(1+r_d)^{N-(t-1)}}\right]\right)}{[1+r_d]^t}\right] + \left[\frac{PAR}{[1+r_d]^N}\right]$$

APPENDIX II - Tables

TABLE 1
Cost of Debt Capital using the Straight-Line Method of Amortization

Coupon Period	**Book Value of Bond Issue**	**Amortization of Discount for period**	**Interest Expense Recorded**	**Before-Tax Cost of Debt (r_d)**	**After-Tax Cost of Debt r_d(1-T)**
0	$ 850,000				
1	865,000	$ 15,000	$ 75,000	8.8235 %	5.2941 %
2	880,000	15,000	75,000	8.6705 %	5.2023 %
3	895,000	15,000	75,000	8.5227 %	5.1136 %
4	910,000	15,000	75,000	8.3799 %	5.0279 %
5	925,000	15,000	75,000	8.2418 %	4.9451 %
6	940,000	15,000	75,000	8.1081 %	4.8649 %
7	955,000	15,000	75,000	7.9787 %	4.7872 %
8	970,000	15,000	75,000	7.8534 %	4.7120 %
9	985,000	15,000	75,000	7.7320 %	4.6392 %
10	1,000,000	15,000	75,000	7.6142 %	4.5685 %
Averages of the straight-line method:				**8.1925 %**	**4.9155 %**
Solution to the traditional method established in Example 1:				**8.2619 %**	**4.9572 %**

TABLE 2
Cost of Debt Capital using the Effective Interest Method of Amortization

Coupon Period	**Book Value of Bond Issue**	**Amortization of Discount for period**	**Interest Expense Recorded**	**Before-Tax Cost of Debt (r_d)**	**After-Tax Cost of Debt r_d(1-T)**
0	$ 850,000.00				
1	860,226.34	$ 10,226.34	$ 70,226.34	8.2619 %	4.9572 %
2	871,297.58	11,071.24	71,071.24	8.2619 %	4.9572 %
3	883,283.52	11,985.94	71,985.94	8.2619 %	4.9572 %
4	896,259.72	12,976.20	72,976.20	8.2619 %	4.9572 %
5	910,308.01	14,048.29	74,048.29	8.2619 %	4.9572 %
6	925,516.95	15,208.94	75,208.94	8.2619 %	4.9572 %
7	941,982.45	16,465.50	76,465.50	8.2619 %	4.9572 %
8	959,808.32	17,825.87	77,825.87	8.2619 %	4.9572 %
9	979,106.94	19,298.62	79,298.62	8.2619 %	4.9572 %
10	1,000,000.00	20,893.06	80,893.06	8.2619 %	4.9572 %
Solution to the traditional method established in Example 1:				**8.2619 %**	**4.9572 %**

TABLE 3
Comparison of the Straight-Line Method with the Effective Interest Method
Original Issue Discount Bond

Coupon Period	Interest Expense Recorded: Straight-Line Method	Interest Expense Recorded: Effective Interest Method	Difference in Expense if Straight-Line is chosen	Present Value of Difference at 8.2619 %
1	$ 75,000	$ 70,226.34	$ 4,773.66	$ 4,409.36
2	75,000	71,071.24	3,928.76	3,352.00
3	75,000	71,985.94	3,014.06	2,375.34
4	75,000	72,976.20	2,023.80	1,473.21
5	75,000	74,048.29	951.71	639.92
6	75,000	75,208.94	(208.95)	(129.77)
7	75,000	76,465.50	(1,465.50)	(840.73)
8	75,000	77,825.87	(2,825.87)	(1,497.43)
9	75,000	79,298.62	(4,298.62)	(2,104.01)
10	75,000	80,893.06	(5,893.06)	(2,664.30)
Total Present Value of Choosing Straight-Line Method over the Effective Interest Method when Coupon Bonds are Originally Issued at a Discount:				**$ 5,013.59**

TABLE 4
Comparison of the Effective Interest Method with the Straight-Line Method
Original Issue Premium Bond

Coupon Period	Interest Expense Recorded: Straight-Line Method	Interest Expense Recorded: Effective Interest Method	Difference in Expense if Straight-Line is chosen	Present Value of Difference at 7.1347%
1	$ 80,000	$ 85,616.33	$ (5,616.33)	(5,242.31)
2	80,000	84,590.10	(4,590.10)	(3,999.10)
3	80,000	83,490.66	(3,490.66)	(2,838.68)
4	80,000	82,312.76	(2,312.76)	(1,755.54)
5	80,000	81,050.83	(1,050.83)	(744.53)
6	80,000	79,698.87	301.13	199.15
7	80,000	78,250.44	1,749.56	1,079.98
8	80,000	76,698.68	3,301.32	1,902.16
9	80,000	75,036.20	4,963.80	2,669.58
10	80,000	73,255.11	6,744.89	3,385.89
Total Present Value of Choosing Straight-Line Method over the Effective Interest Method when Coupon Bonds are Originally Issued at a Premium:				**$ (5,343.40)**