

Daily vs. monthly rebalanced leveraged funds

William Trainor Jr.
East Tennessee State University

ABSTRACT

Leveraged funds have become increasingly popular over the last 5 years. In the ETF market, there are now over 150 leveraged funds with \$30 billion in assets. Fund companies have continued to innovate this product by leveraging both bullish and bearish calls on a plethora of indexes. In addition, the leverage available now ranges from -3.0x to 3.0x times the underlying index return. The major drawback to these funds for longer-term investors is a compounding issue due to daily rebalancing which causes the specified leverage to generally decay over time.

In answer to this problem, monthly as opposed to daily rebalanced leveraged funds are now available with more on the way. This study examines the characteristics of monthly relative to daily rebalanced leveraged funds in relation to the magnitude of the leverage, trend, time, volatility, and return. Although the monthly leverage funds clearly have advantages over daily leveraged funds for the intermediate term investor, the advantages dissipate substantially beyond six months. Thus, for long-term buy and hold investors, monthly rebalanced leveraged funds have the same drawbacks as daily leveraged funds.

Key Words: Leveraged ETFs, monthly leveraged funds, beta drift

INTRODUCTION

Although the first leveraged fund was introduced in 1993, it has been the advancement of these funds within the ETF market that has made them popular. There are now more than 150 leveraged and inverse ETFs with total assets of \$30 billion, (Spence, 2010). Until recently, leveraged funds have focused on daily multiples ranging from -3.0x to 3.0x on a particular underlying index. For the uninitiated reader, funds with a positive multiple are referred to as bullish funds while those with a negative multiple are referred to as bearish funds. As an example, a bearish investor who holds a leveraged fund with a -2.0x multiple would expect to earn two times the inverse of the daily return of the underlying index. If the index fell 1% on a particular day, the investor holding this fund should earn 2%. A 2.0x bullish fund on this index would lose 2%.

The most pertinent feature of these leveraged funds which the fund companies have endeavored to make clear is that they are only designed to attain a multiple of the underlying index return on a daily basis. Due to compounding issues, often referred to as the Constant Leverage Trap, this daily multiple cannot be guaranteed with any certainty into the future.

To demonstrate this more concretely, assume an investor knows with certainty that the underlying index will be up between 8 to 12% over one year. The market standard deviation is 40%. Should the investor A) simply invest in the index, B) invest in a 2.0x daily leveraged fund, or C) invest in a 3.0x daily leveraged fund? The answer is surprisingly "A" with a guaranteed return between 8 and 12%. The average return for a 2.0x leveraged fund is 3.1% with a minimum of -5.0% and a maximum of 10.5%. The 3.0x leveraged fund has an average return of -17.7% with a minimum of -32.5% and a maximum of -3.7%. Results are based on 20,000 simulations. This is the compounding problem for leveraged funds.

In fact, virtually any realized multiple is possible for any leveraged fund given enough time, return trend, and volatility. This multiple can work both for and against the investor, but under most market conditions as evidenced above, the effects of volatility generally cause long-term return multiples to be much less than a fund's daily leverage ratio and on occasion, a bullish or bearish leveraged fund can lose money over an extended period of time even if the underlying index is up or down respectively over that same time period. This has led to less than favorable press for these funds, (Justice 2009; Lauricella 2009; Spence 2009; Zwiig 2009).

Several academic studies have also pointed out how these leveraged fund's multiples decay over time, (Trainor & Baryla 2008, Carver 2009, Avellaneda & Zhang 2009, Lu, Wang, & Zhang 2009). The message apparently has been received as the implied holding period for most of these funds is now less than a week. The overriding conclusion is that even under ideal conditions, holding daily rebalanced leveraged funds beyond a few months will almost certainly lead to disappointment in terms of maintaining the leverage ratio.

In answer to this drawback, Direxion changed their daily leveraged mutual funds to monthly leveraged funds in September of 2009. In addition, Direxion filed to offer 40 new inverse and leveraged ETFs that are based on monthly rebalancing. Obviously this eliminates the compounding problem out to one month, but the more pressing question for investors is how advantageous these monthly funds will be for longer term holding periods. Specifically, what are the effects of leverage magnitude, trend, time, and volatility for these funds relative to daily leveraged funds? Finally, monthly leveraged funds create an interim investment issue in which the initial specified leverage can only be guaranteed if the fund is purchased when the fund is

rebalanced each month. This study addresses these issues which investors in these funds must now contemplate.

BETA DRIFT AND DECAY

For this study, beta is defined as the realized cumulative return of a leveraged fund over a particular time period divided by the cumulative return of the underlying index. Mathematically, the underlying index return and leveraged fund return over any time frame can be written as:

$$X_N/X_0 - 1 = \prod_{i=1}^N (1 + r_i) - 1 \quad (1)$$

$$F_N/F_0 - 1 = \prod_{i=1}^N (1 + Lr_i) - 1 \quad (2)$$

where X represents the underlying index, F is the leveraged fund, L is the daily or monthly leverage ratio, and r_i is the return for each rebalanced period. The realized beta for any time period is simply:

$$\text{Beta} = (F_N/F_0 - 1)/(X_N/X_0 - 1) \quad (3)$$

Beta drift (BD) can be thus be defined as:

$$\text{BD} = \text{Beta} - L \quad (4)$$

Beta drift refers to the fact that over time, the beta of a leveraged fund tends to deviate from its defined daily or monthly leverage ratio. The term beta decay is often used since the realized leverage ratio usually falls over time for buy and hold investors. However, this is not a given, as beta drift depends on the relative effects of the leverage magnitude, time, the underlying return trend, and volatility.

Leverage and Trend

To demonstrate the value of trend, Figures 1 and 2 show the rolling three-month returns (not annualized) and annualized standard deviations based on daily returns for the S&P 500 Index from January 1, 2000 to June 30, 2010. In addition, Figure 1 superimposes the number of 3-month periods a 3.0x leveraged daily fund's beta would be greater than 100% of the given leverage ratio and Figure 2 superimposes the same data for a monthly leveraged fund.

As one can discern from the graph, volatility has been relatively stable around the 20% range until the sharp spike due to the financial crisis where it reached almost 80%. The 3-month return has shown some fairly dramatic short-term trends ranging from -40% to 40%. Both Figures 1 and 2 demonstrate the value of trend. High positive (negative) trend coupled with low volatility actually causes the beta for leveraged bullish (bearish) funds to increase. Both Figures 1 and 2 show that in general, the 3-month cumulative betas exceed the initial leverage ratio of 3.0x only when the trend is highly positive in periods of low volatility. In addition, it is clear the monthly leveraged fund achieves this feat to a much greater extent than the daily leveraged fund.

Table 1 defines this exactly and shows the number of periods out of a combined overlapped total of 2,639 that a 2.0x, 3.0x, -2.0x, and -3.0x daily and monthly leveraged fund

would have exceeded 100% of the leverage. For the daily long funds, 16 to 17% of the time the beta or realized leverage over a 3-month period was greater than the daily leverage. For the monthly leverage, this number increased to almost 28%. As expected, even with the 2001 market crash and 2008 financial crisis, the bearish funds did not perform as well. With a daily leveraged fund, the chance the trend increased the realized leverage when the market was moving favorably for the investor was only 5 to 6% of the time while the monthly leveraged fund increased this probability to 15%. Thus, at three months, clearly the monthly leveraged funds are advantageous to longer term investors. A statistical test confirms that the differences between the monthly and daily leveraged funds are significant.

Leverage, Volatility and Time

Although positive (negative) trend can increase the beta over time for bullish (bearish) funds, volatility has the exact opposite effect and dominates over any extended period of time. Employing Monte Carlo simulation, Figure 3 shows the general relationship between beta and volatility for a leveraged fund based on 10,000 simulations. In this example, as volatility increases from 20% to 80%, the probability that the beta of a 3.0x daily fund remains within 20% of 3.0x after 6 months falls from 55% to 17%. Although not shown, for the monthly leveraged fund, this probability falls from 63% to 20%. Thus, it is true the monthly leveraged fund provides some added certainty for the longer term investor, but this advantage becomes less pronounced at very high volatility levels.

Figure 4 shows the general relationship of how volatility and time affects the beta of a 2.0x daily and monthly leveraged fund. For both funds, the probability the leverage ratio will remain within 20% of 2.0x by 6 months falls to less than 80% and this is for a “normal” market, (10% annual expected return and 20% standard deviation is assumed). By one year, there is virtually no difference between the two funds. The same general relationship holds for other leverage ratios.

Table 2 Panel A shows the probabilities that a daily and monthly leveraged fund’s beta will remain within 20% of its initial leverage ratio for a 2.0x, 3.0x, -2.0x, and -3.0x fund. Time periods out to one year are examined under the assumption of a 10% expected return and 20% standard deviation.

At one month, clearly the monthly leveraged funds dominate as there is a 100% chance their betas will be equal to their initial leverage ratio. It is also apparent that the 3.0x and -3.0x daily funds have a much smaller probability of remaining close to their initial leverage ratios relative to the -2.0x and 2.0x funds, 90% to 81% and 73% to 65% respectively.

Furthermore, the bearish funds do much worse than the bullish funds and this is not just because the market is assumed to trend upward. Inverse funds by their design have both a base and leverage issue. That is, even a non-leveraged inverse fund cannot maintain the cumulative inverse return of the underlying index. To see this, consider an index that starts at 100 and falls 10% in period 1, then another 10% in period 2, in effect going from 100 to 90 to 81. The inverse fund would increase from 100 to 110 to 121. Thus, even though it is not a levered fund per se, the inverse fund is up 21% over the two periods while the index is only down 19%. This is the base issue and causes the cumulative beta to drift from the initial leverage ratio even for -1.0x inverse funds. This additional effect generally causes leveraged inverse funds to lag the beta performance of their bullish fund counterparts.

By 6-months, the difference between the daily and monthly leveraged fund is approximately five percentage points and by one year, the difference averages less than two percentage points. From an investing standpoint, regardless of the leverage, the distinction between daily and monthly rebalanced funds becomes relatively blurred after 6 months and by one year there is little economic or quantitative difference between the two funds. This is not to say that investors should hold even monthly funds for 6 months since the probability they remain close to their initial leverage ratios is only 80% for a 2.0x and 40% for a -3.0x.

Under more volatile conditions, these numbers are even worse as Table 2 Panel B shows. Assuming a 40% standard deviation, the probability that leverage remains within 20% of its initial leverage for a daily fund after 6 months ranges from 56% for a 2.0x and only 17% for a -3.0x. For the monthly fund, these probabilities range from 64% to 21%. By one year, none of the funds have even a 50% chance of remaining within 20% of their initial leverage ratio. Thus, for long-term investors, beta decay is a serious issue whether daily or monthly leveraged funds are used, especially when one considers that the risk does not similarly decay.

Compounding Error

To economically demonstrate the compounding problem, Table 3 shows the compounding error based on 20,000 simulations under the assumption of a 10% expected return and 20% or 40% standard deviation for 6 months and 1 year. The compounding error for positive leveraged funds is defined as:

$$(F_N/F_0 - 1) - L*(X_N/X_0 - 1) \quad (5)$$

In words, this is the leverage return minus $L * \text{index return}$ where L is the stated leverage of the underlying fund. For short funds, the compounding error is defined as leverage fund return plus $L * \text{index return}$ where L is the stated leverage of the underlying fund. In this way, negative numbers are associated with returns less than expected due to the compounding issue, and any positive returns would be associated with excess return when compounding actually works for the investor. Median numbers are reported as the mean values are skewed due to a few instances of very large values.

As Table 3 shows, with a 20% standard deviation, by 6 months the median daily 2x rebalanced leverage fund has a compounding error of -1.04%. This means if the market was up 5%, rather than being up 10%, the 2.0x leveraged fund is only up 8.96%. The 2.0x monthly fund shows its advantage here as it only has a -0.44% compounding error. By one year, the error for a 2x daily is -2.10% while the error for the 2.0x monthly fund has increased to -1.37%. These errors become particularly pronounced with the 3.0x funds and are quite dramatic for bearish funds especially when the market ends up negative with compounding errors exceeding 20% on average. Overall, the compounding errors are not economically horrific so to speak assuming an investor experiences a fairly normal market.

However, as the annualized standard deviation is increased to 40%, the median compounding errors quickly become quite large for all the leveraged funds reaching -73.72% for the -3.0x daily rebalanced fund in a down market. During the financial crisis, even the popular press had several stirring accounts of short funds losing money even though the market was down, (Zweig, 2009 for example).

Thus, with high market volatility, both daily and monthly funds have significant compounding error beyond 6 months. For example, by one year the 2.0x daily fund has a compounding error of -14.39%. Even if the market is up 15%, rather than being up 30%, the investor is only up 15.61%.

Table 4 highlights this in absolute terms by showing the average return associated with the funds for when the market is up 0 to 10%, 10 to 20%, and 20 to 30% assuming a 10% expected return and 40% standard deviation given a one year holding period. Results are based on an initial run of 20,000 simulations. For the daily and monthly 2.0x funds, the average return when the index averages 5% is a -6.3% and -5.5% respectively. It is not until the market is assumed to increase by 25% do the leverage funds even outperform the index return. Thus, any period of high volatility deteriorates returns for both daily and monthly rebalanced funds to such a degree that their returns are even less than the underlying index they are supposed to magnify. For an investor unfortunate enough to invest in leveraged bearish funds when the market increases, average losses exceed 40% even when the market is only up 5%.

INTERIM INVESTORS

A final issue that must be considered before investing in a monthly leveraged fund is to be aware of the fund's rebalancing date. The monthly leverage is only exact for one particular starting date. For long funds, if the index is above the initial start value that month, the realized leverage will be less than the underlying fund's stated leverage due to the higher base that one is starting with. If the index is less than the initial start value, the realized leverage will be greater than the stated leverage.

For short funds, just the opposite holds. That is, if the index is above the initial start value, the realized leverage will be greater than stated due to the higher base that one is starting with. If the index is less than the initial start value, the realized leverage will be less than stated.

Table 5 demonstrates this issue for a 3.0x and -3.0x fund. Assume the underlying index increases from 100 to 105, then either moves to 110 or 96. It is assumed the investor buys into the monthly fund at time $t = 1$. If the investor buys in at 105 and the index increases to 110, leverage will end up being $13.04\%/4.76\% = 2.74$. If the index falls to 96 instead, leverage will be $-23.48\%/-8.57\% = 2.74$. Regardless where the index ends up, the leverage ratio for the 3.0x fund will be 2.74x if the leveraged fund is purchased when the underlying index is 5% greater than when the monthly fund initially reset.

In the same manner, the leverage will be -3.71x for the -3.0x fund if an investor buys in at time $t=1$. If at $t=1$, the index is below 100, the 3.0x beta will have a interim beta greater than 3.0 and the -3.0x fund will have an interim beta less than -3.0 in absolute value. These results are due entirely to the base issue. In this case, the base for the interim investor is 5% above the initial reset value for the leveraged fund.

Appendix 1 mathematically derives what the actual leverage will be for any circumstance and the final equation is:

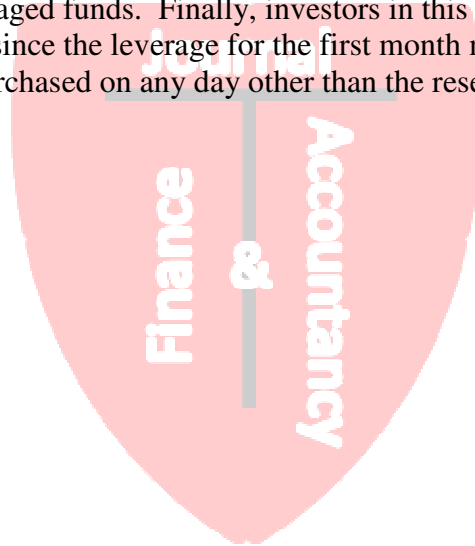
$$\text{Beta} = (L-1)/(1+Lr_t) + 1 \quad (6)$$

where L is the initial leverage ratio and r_t is the return of the underlying index from when the fund is rebalanced to when the fund is actually purchased within the rebalancing period.

CONCLUSION

Leveraged funds continue to innovate. However, despite the plethora of indexes and leverage ratios one can now choose from, the daily rebalancing feature which results in increasing beta decay over time remains a major drawback for investors with longer term horizons. In response to this, monthly leveraged funds are now available with more on the way. The advantage of a monthly leverage fund is that the leverage is now certain for time frames up to one month. As expected, the decay of monthly leverage funds is much less than daily leverage funds, but it appears for most market conditions, the realized leverage ratio between a daily and monthly leveraged fund with the same initial leverage becomes fairly blurred after 6 months.

Risk averse long-term investors will likely find this expected beta decay unacceptable given that the riskiness of these funds. When the market is relatively volatile, both daily and monthly leveraged funds will likely underperform just holding the underlying index. Thus, even monthly leveraged funds should not be considered for the typical buy and hold investor. For intermediate-term or active investors, monthly leveraged funds do have added value and are a nice alternative to daily leveraged funds. Finally, investors in this market need to be aware of the interim investment issue since the leverage for the first month may be much different from what they are expecting if purchased on any day other than the reset day.



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Appendix 1: Intra-month Leverage Derivation

Mathematically, given any initial index value and the value of the index when one buys into the monthly fund, the leverage realized from that date forward to the end of the month can be calculated as follows:

Let $X_N = X_0(1 + r_N)$ and also $X_N = X_t(1 + r_2)$, and $X_t = X_0(1+r_1)$

where the subscript 0 is for the beginning of the month, time t which can be anytime within the month, and n is at the end of the month. X is the underlying index and r is for the return. There will be an r_1 , r_2 , and a r_N to designate the return from $t = 0$ to $t = 1$, from $t = 1$ to $t = N$, and from $t = 0$ to $t = N$.

Letting L stand for the monthly leverage, then a levered fund value at time t and N assuming no deviations from a continuous leverage value equal to the funds leverage throughout the month will be

$$LX_t = [L (X_t/X_0 - 1) + 1] * X_0$$

$$LX_N = [L (X_N/X_0 - 1) + 1] * X_0$$

A levered fund's return (LR) from time t to N will be

$$LR = LX_N/LX_t - 1$$

A levered fund's interim beta or implied leverage for investors not buying at time = 0 will be

$$\text{Beta} = LR/r_2$$

Substituting terms, one finds that the realized beta for an interim investor will be strictly dependent on the value of the index at time t relative to time 0 and can be rewritten as

$$\text{Beta} = (L-1)/(1+Lr_t) + 1$$

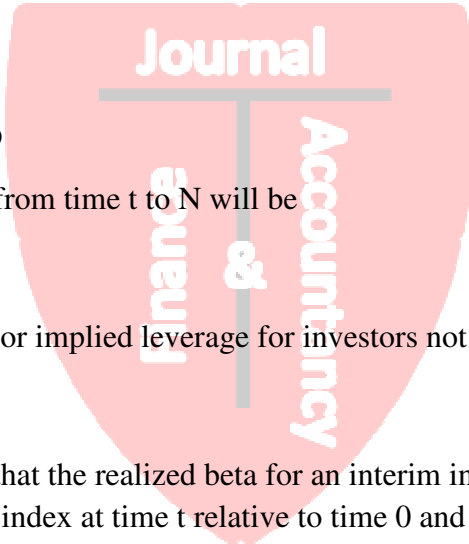


Figure 1

3-month periods in which a 3.0x beta is greater than 3.0 for a daily leveraged fund. Time periods are delineated by columns equal to 3.0 when this occurs.

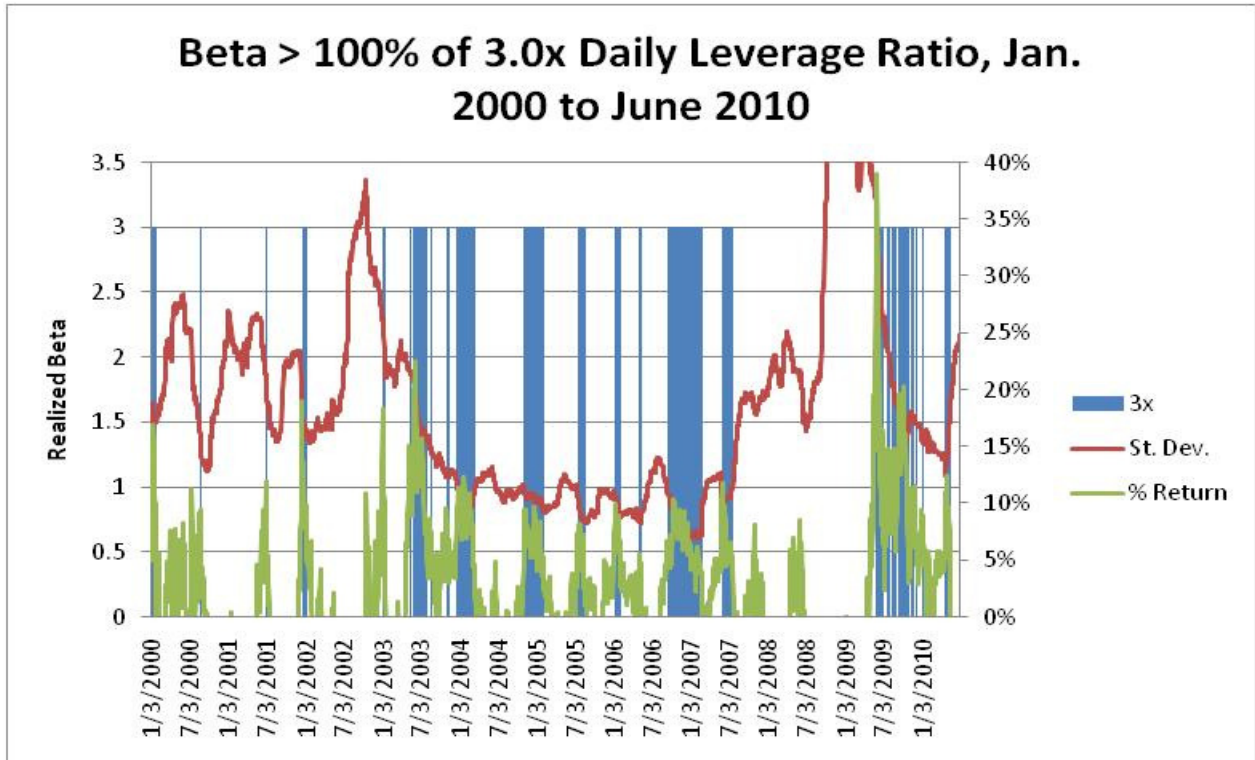


Figure 2

3-month periods in which a 3.0x beta is greater than 3.0 for a monthly leveraged fund. Time periods are delineated by columns equal to 3.0 when this occurs.

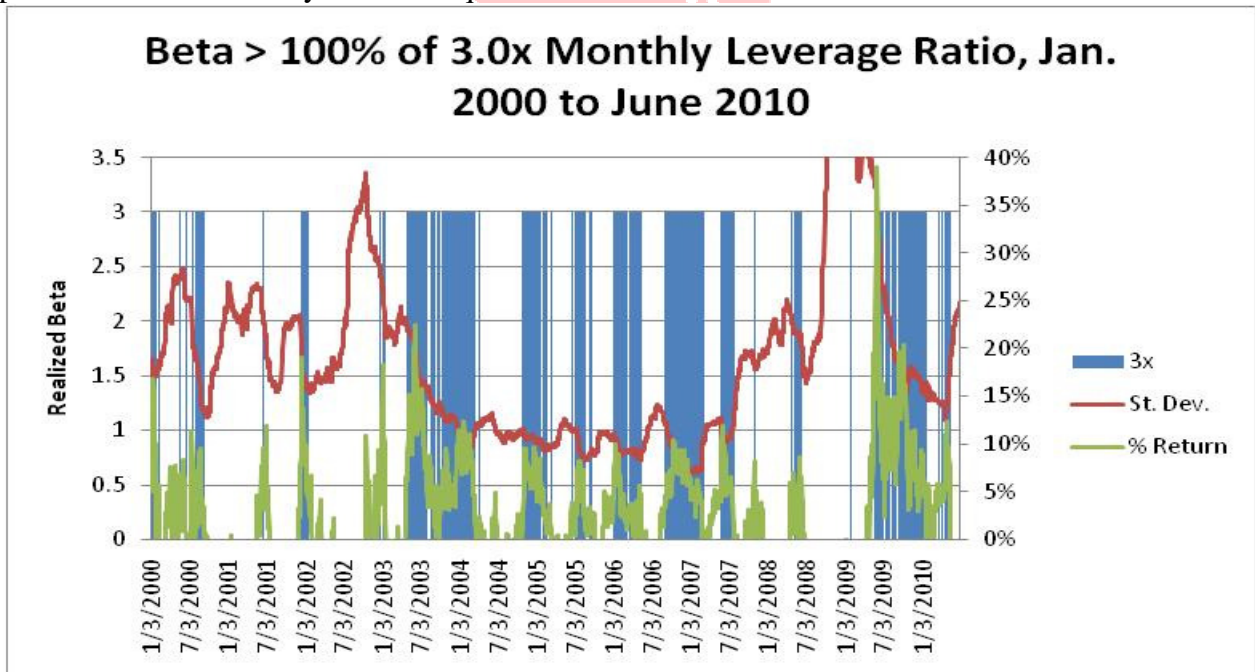


Figure 3

At 6-months, the probability that beta for a 3.0x daily leveraged fund will be within certain values is shown for standard deviations ranging from 20 to 80%. Annualized expected return is set at 10% and data is derived based on 10,000 runs using Monte Carlo simulation..

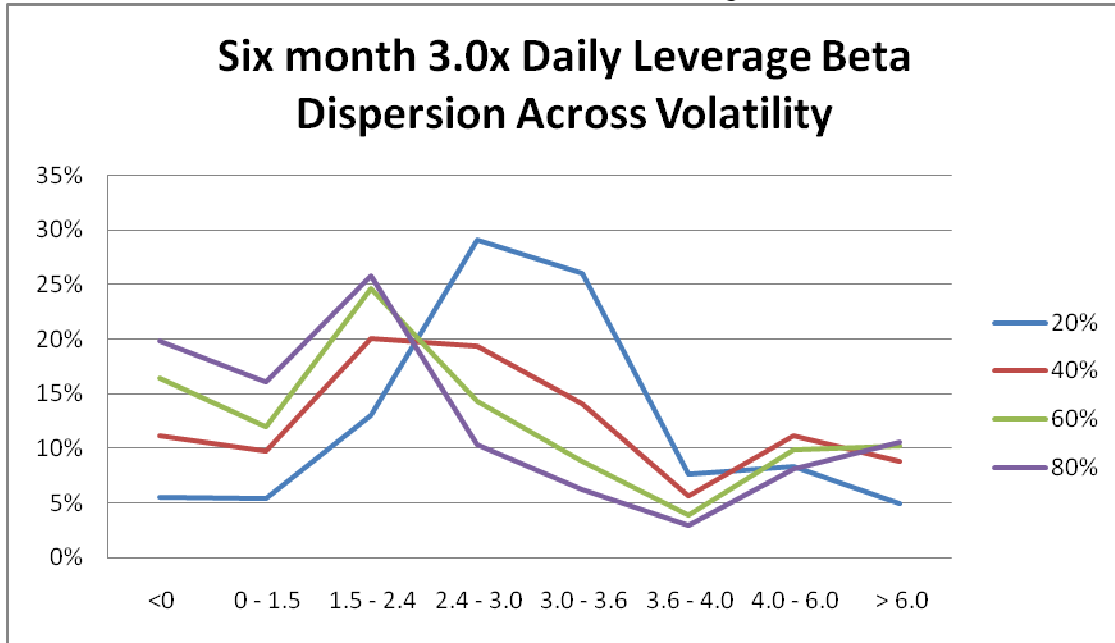


Figure 4

Probability that the beta will be within 20% of 2.0x for a daily and monthly leveraged fund assuming a annual 10% expected return and 20% standard deviation.

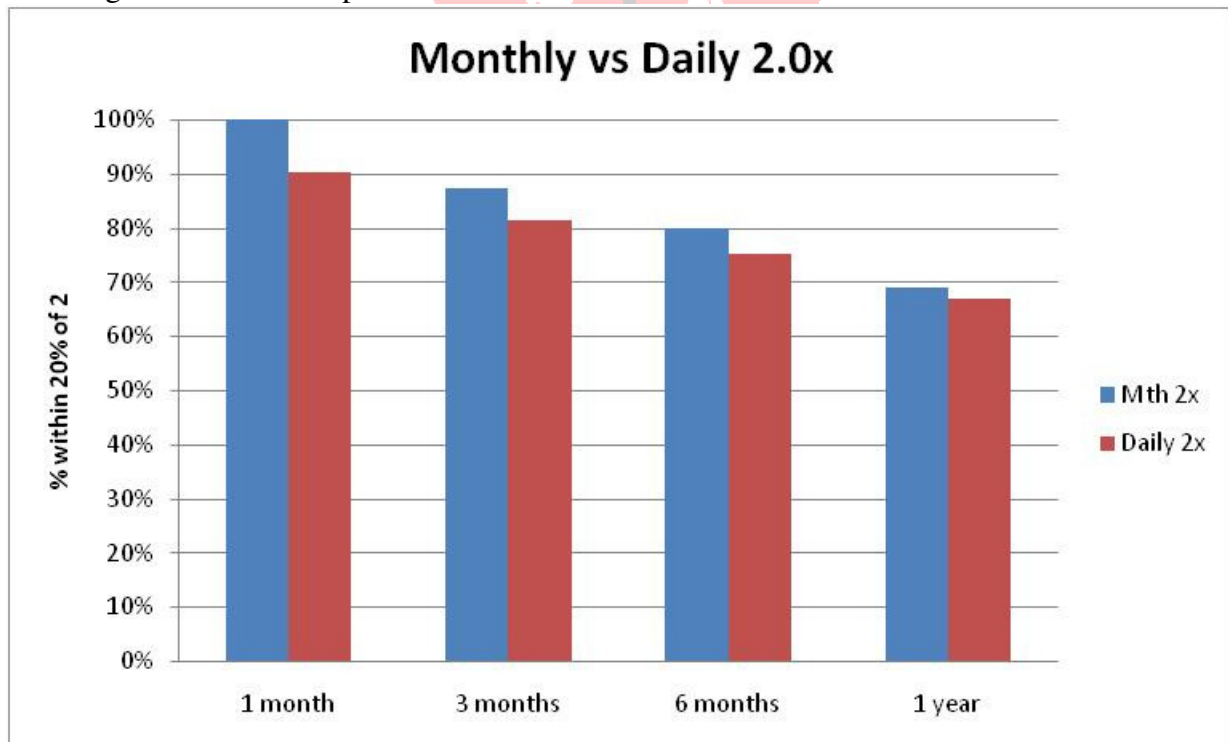


Table 1

There are 2,639 rolling 3-month periods between January 2000 and June 2010. The number of positive compounding results for 2.0x, 3.0x, -2.0x, and -3.0x for daily and monthly levered funds are given.

Total Number of Periods = 2,639	Number of periods beta > 100% of leverage at 3 months	
	Daily	Monthly
2.0x	449 (17.0%)	735 (27.9%)*
3.0x	430 (16.3%)	731 (27.7%)*
-2.0x	163 (6.2%)	414 (15.7%)*
-3.0x	137 (5.2%)	409 (15.5%)*

*Statistically significantly greater than daily counterpart at the 5% level.

Table 2

Probability that a daily or monthly fund's beta will remain within 20% of its initial leverage ratio from one month to one year. Results are based on 10,000 simulations using an annual expected return of 10% and standard deviation of 20% for Panel A and standard deviation of 40% for Panel B.

Panel A. $E(R) = 10\%$, $\sigma = 20\%$

Leverage	1 month		3 months		6 months		1 year	
	Daily	Monthly	Daily	Monthly	Daily	Monthly	Daily	Monthly
2.0x	90%	100%*	81%	87%*	75%	80%*	67%	69%*
3.0x	81%	100%*	66%	78%*	55%	63%*	43%	46%*
-2.0x	73%	100%*	54%	70%*	43%	50%*	32%	34%*
-3.0x	65%	100%*	45%	62%*	33%	40%*	24%	26%*

Panel B. $E(R) = 10\%$, $\sigma = 40\%$

Leverage	1 month		3 months		6 months		1 year	
	Daily	Monthly	Daily	Monthly	Daily	Monthly	Daily	Monthly
2.0x	80%	100%*	67%	78%*	56%	64%*	45%	48%*
3.0x	64%	100%*	45%	61%*	33%	39%*	25%	26%
-2.0x	50%	100%*	32%	47%*	22%	28%*	16%	17%
-3.0x	41%	100%*	25%	38%*	17%	21%*	11%	13%*

*Statistically significantly greater than daily counterpart at the 5% level.

Table 3

Median compounding error for daily and monthly fund returns. Error is defined as actual leverage return minus (plus) leverage times index return for positive (negative) leverage. Results are based on 20,000 simulations. Market returns are separated by whether the index increased or decreased for the sub-period stated.

Panel A. $E(R) = 10\%$, $\sigma = 20\%$

Leverage	6 months (Market up)		1 year (Market up)		6 months (Market down)		1 year (Market down)	
	Daily	Monthly	Daily	Monthly	Daily	Monthly	Daily	Monthly
2.0x	-1.04%	-0.44%*	-2.10%	-1.37%*	4.90%	-0.57%*	-2.06%	-1.56%*
3.0x	-3.65%	-1.50%*	-7.88%	-5.09%*	-2.63%	-1.53%*	-5.05%	-4.02%*
-2.0x	-0.99%	-0.52%*	-0.01%	-0.04%	-4.51%	-2.31%*	-9.92%	-7.16%*
-3.0x	-1.04%	-0.62%	2.41%	1.73%	-9.50%	-4.94%*	-20.48%	-15.24%*

Panel B. $E(R) = 10\%$, $\sigma = 40\%$

Leverage	6 months (Market up)		1 year (Market up)		6 months (Market down)		1 year (Market down)	
	Daily	Monthly	Daily	Monthly	Daily	Monthly	Daily	Monthly
2.0x	-6.29%	-2.90%*	-14.39%	-9.71%*	13.54%	-1.48%*	-3.58%	-3.12%
3.0x	-20.69%	-10.30%*	-44.67%	-34.47%*	-4.35%	-3.08%	-3.09%	-3.76%*
-2.0x	-3.40%	-2.73%	-0.26%	-1.61%*	-18.68%	-9.71%*	-38.61%	-29.52%*
-3.0x	-0.54%	-1.92%*	12.96%	8.72%*	-38.53%	-21.72%*	-73.72%	-63.84%*

*Statistically significantly different than daily counterpart at the 5% level based on Mann-Whitney test.

Table 4

Average annual returns for daily and monthly leveraged fund returns given the market ends up between 0 to 10%, 10 to 20%, and 20 to 30% after one year. Results are based on 20,000 simulations.

$E(R) = 10\%$, $\sigma = 40\%$

Leverage	Index up 0 to 10%		Index up 10 to 20%		Index up 20 to 30%	
	Daily	Monthly	Daily	Monthly	Daily	Monthly
Index	4.86%		14.92%		24.84%	
2.0x	-6.26%	-5.47%*	12.72%	14.09%*	33.00%	34.68%
3.0x	-28.69%	-28.16%	-5.76%	-3.31%*	20.81%	25.04%*
-2.0x	-43.65%	-44.50%*	-52.98%	-54.44%*	-60.21%	-62.60%*
-3.0x	-66.80%	-71.01%*	-74.69%	-79.19%*	-80.32%	-85.40%*

*Statistically significantly different than daily counterpart at the 5% level.

Table 5

Example of an interim investor within a monthly rebalanced fund not attaining the initial leverage multiple. Investor is assumed to buy in at $t = 1$.

	t = 0	t = 1	t = 2a	t = 2b
Index	100	105	110	96
Index Return		5.00%	4.76%	-8.57%
Cumulative Index Return		5.00%	10.00%	-4.00%
3.0x Return			13.04%	-16.98%
3.0x Beta			2.74	2.74
-3.0x Return			-17.65%	31.76%
-3.0x Beta			-3.71	-3.71