Math CAMMP:
A constructivist summer camp for teachers and students

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ABSTRACT

A summer session, math methods course for elementary teachers incorporates 30 hours of instruction that emphasizes (1) developmentally appropriate instructional strategies, (2) hierarchical levels of increasingly abstract manipulatives, (3) ongoing assessment of student learning, (4) integrated thematic instructional modules, (5) team planning and teaching, and (6) web- and server-based math software. The course culminates in a math camp for rising first through sixth grade students who come to the university campus. The student part of math camp is a week long, full day math and robotics camp, with the math portion delivered each morning by master's level teachers enrolled in the graduate course. The integration of pedagogical components with supervised teaching comprises a type of "guided constructivism" not found in other graduate classes in the master's program.

Keywords: constructivism, elementary school math, instructional strategies, manipulatives

INTRODUCTION

The Comprehensively Applied Manipulative Mathematics Program (CAMMP) began in 1991 as an idea for teaching elementary and middle school teachers constructivist teaching strategies for math. This graduate course is offered only during summer session, and it entails a novel two-for-one enterprise. First, elementary and middle grades teachers enroll in a summer-term graduate course on teaching methods for mathematics. Second, using the strategies they have learned, teachers then implement a half-day, week-long summer camp for rising first through sixth grade students who come to campus.

This two-for-one enterprise has been successful for two decades because teachers continue to seek new strategies for "hands-on learning," something CAMMP has a reputation for delivering. Additionally, the course works because it offers families living near the university extra enrichment opportunities for elementary and middle grades students to get an early start on concept building for next year's mathematics curriculum.

MATH CAMMP FOR TEACHERS

There exists a perception that education coursework is both too easy and ineffectual as a way to train teachers for managing day-to-day instruction in modern classrooms. In fact, Willoughby (1990) has pointed out the extreme dependence of elementary teacher on textbooks for the content, sequence, and timing of instruction. We suspect that too many modern classrooms are today much as Willoughby found them two decades ago. But, there is an added concern here with instruction. That is, elementary teachers have a measurable tendency to teach the way they were taught in elementary school, regardless of the education courses they may have taken for licensure in their state (Bauersfeld, 1998; Désautels, 2000). These considerations led the authors to design a math methods course that transcended traditional "note and hope" coursework wherein students took copious notes and instructors hoped they would be put to use in the classroom. That is, to be truly effective, a math methods course had to incorporate elements of innovation, practice, demonstrated usage with real students, and feedback from the instructors.

The graduate methods course described in this paper is structured for 45 contact hours and is co-taught annually by two faculty during a summer session. The first 30 hours are spent in class reconstructing K-6 mathematics concepts and procedures with teachers using a constructivist, hands-on approach. The approach is termed "guided constructivism" (cf. Freudenthal, 1991; Green, Piel, & Flowers, 2008; Piaget, 1974). Accordingly, teachers do not need to wait for their students to re-invent, for example, the standard long division algorithm; it has already been invented and is quite efficient. Rather, teachers are encouraged to incorporate a variety of strategies for solving division problems with manipulatives. These manipulative strategies allow students to develop deep conceptual understanding of how the long division algorithm works.

The various teaching strategies taught in the course constitute the elements of CAMMP. As collection of teaching strategies, these elements have been around for some time. But in the CAMMP approach to teaching mathematics, the elements get

reorganized into a coherent, hierarchical, constructivist model of teaching (Piel & Green, 2010).

Elements of CAMMP

Problem Solving Modules. An essential strategy for learning, problem solving is the primary motivation for learning mathematics, and instruction begins with problem solving situations (NCTM, 2000). Grade level teams design integrated, thematic modules of problems reflecting appropriate grade-level content, such as: insect fractions, rain forest multiplication, arctic subtraction, and space shuttle decimals.

Action Language. CAMMP teaching strategies incorporate a variety of hands-on objects. Traditional language for operations doesn't really tell children what arrangements are being formed or even how symbols can be arranged. As indicated in Table 1 (Appendix), action language describes operations in terms of actions for arranging objects (where these actions are later extended to symbols).

Concrete Manipulatives. When new concepts are introduced, concrete manipulatives should be used, whether students are in first grade or high school. Counters and Cuisenaire rods are used frequently, because both entail one-to-one correspondence between objects and their value, and because they can be held in the hand. With these manipulatives, objects can be joined, taken away, arranged in sets, and grouped into –action language used in both arithmetic and in prealgebra.

Representational Manipulatives. These are one level more abstract than concrete manipulatives and allow children to extend actions on objects to actions on representations. Representational manipulatives may be pictures, tally marks, or base 10 blocks. Manipulatives at this level reflect many-to-one values for the objects being manipulated. For example, a tally mark may count as a one, a ten, or a hundred; the same mark has multiple values depending on which column it occupies. Parents are amazed that their children learn to solve multi-digit problems using these manipulatives long before they have memorized basic facts.

Technology. Teachers incorporate into their thematic modules one hour each morning of computer-based math activities. These may be undertaken on either internet web sites or the college's network-based math software.

Transition Activities. Many teachers move directly from hands-on manipulatives to symbolic activities as if this conceptual leap was somehow transparent to students. From a student's point of view, understanding place value often gets lost in this transition, and it is very difficult to teach using a purely textbook or symbolic approach. The transition activities are the same ones for all the operations – expanded notation and partial sums, products, differences, and quotients – so that the same strategy will be successful for students regardless of which operation they are performing. The partials are particularly effective because they do not require regrouping during multi-digit operations.

Symbolic Algorithms. Teachers learn that symbolic algorithms should come at the end of a sequence of developmentally appropriate actions that give meaning to the arithmetic operations and that originate with actions on objects. By the time CAMMP students learn basic facts and perform standard algorithms, they have a rich history of problem solving actions upon which symbolic meanings can be constructed.

Alternative Algorithms. Some students exhibit a history of difficulty with certain algorithms. For these students, teachers learn several alternative algorithms that require less cognitive demand than the standard algorithm.

Quantitative Solutions. These are the numerical "answers" to math problems and are the most familiar element of learning mathematics for teachers. But since real world problems are the beginning of instruction, quantitative solutions are incomplete. They need to be linked back to the objects they represent.

Referential Meanings. These are the objects to which quantitative solutions refer. Whether referential meaning is trees, bicycles, glaciers, or onions, this meaning can only be obtained by referring to the objects in a problem solving situation. If the is no problem to solve, no situational context, then there is no referential meaning for a solution.

Checking Reasonableness. At the transitional and symbolic levels students should learn that a complete solution includes an assessment of its reasonableness. Upper and lower limits are used to establish the reasonableness of a computed solution. For every problem, students determine the logical maximum value and the logical minimum value of the solution. If the computed solution lies between these two logical limits, it is deemed *reasonable.*

Checking Accuracy. Teachers learn a technique for checking accuracy called *Casting Out Nines*. The method is efficient and fun; children often beg teachers, "Can we check our answers, PLEEEEEse?" This method is imperfect because it may occasionally result in errors. Yet, students and teachers like it so much that it has become a basic CAMMP strategy.

Course Structure – Scope, Sequence, and Impact

Math CAMMP incorporates the elements above into instructional planning through the use of integrated, thematic modules. Specific content for the modules may come from science, history, economics, ecology, or other themes, and the theme becomes the organizing idea for generating mathematics problems. Teachers are assigned to grade-level teams that design a five-day module of learning activities. The aim of the course is to produce better math teaching. The week that students come to Math CAMMP is the teachers' laboratory. It is their opportunity to use the strategies they have learned, taught through the content of the thematic module, and supervised by the course instructors.

Along with the integrated, thematic module, teachers integrate math technology through internet-based math games and networked K-6 math software. One hour each morning is spent rotating groups of students through the college's computer labs. The technology component provides representational and symbolic levels of math content for the students at an individualized pace. Teachers report popular URLs to parents in their week end assessments send home to parents.

Research has clearly established the fact that elementary teachers bring to their classrooms consistent, identifiable misconceptions about arithmetic (Ball, 1990; Graeber, Tirosh, & Glover, 1989). In that context, it is interesting to note that the elements used in Math CAMMP produce two consistent outcomes with teachers. First, the CAMMP approach to teaching teachers how to teach math reverses their own arithmetic

misconceptions (Green, Piel, & Flowers, 2008). Second, teachers learn arithmetic meanings, concepts, and relationships that they did not know they didn't know (Green, Piel, & Flowers, 2008). Teachers continue to enroll in the summer Math CAMMP course because word of mouth tells them it is a "career changing" course.

MATH CAMMP FOR KIDS

The last 15 contact hours of the course constitute both the teachers' practicum and the Math CAMMP for Kids. Parents register their children online and pay $230 tuition (ample scholarships allow low income students to participate). Although math is the primary motive for the camp, children attend a full day camp. They spend mornings learning the essential concepts underlying next year's math curriculum in the North Carolina Standard Course of Study. In the afternoon, they do robotics, beginning with hand- and battery-powered machines for primary students and topping out with Lego MINDSTORMS® programming for older students. Robotics teachers are hired by the university and are not participants in the math course.

The student-teacher ratio at Math CAMMP is exceptional. Based on teacher's grade-level interest, they may have as few as two students or as many as five. In 2011, for example, teachers averaged 2 1/3 students. During the math camp week teachers receive immediate, direct feedback from the course instructors, and they compile documentation for each student that conveys to parents (1) assessments of student learning, (2) the teacher impact on student learning, and (3) "work on" activities suggested for the remainder of the summer.

Mondays are typically chaotic, as some students arrive late. Others must be assigned to higher grades, given their math ability. And there are students who arrive without lunches and still must be fed. One Math CAMMP motto is "No Whining!" Teachers understand that changes happen and that no one suffers if everyone commits to maximizing student fun and learning. Given the low teacher-student ratios, it is unsurprising that many teacher volunteer when asked, "Who can take another fourth grader this week?"

The visibility of math campers in the university's child-sterile environment has prompted a steady growth of child friendly offerings by other faculty. Camps on Campus today enrolls more than 600 students in summer enrichment programs initially pioneered by Math CAMMP back in the early 1990s. Still, with all the enrichment camps now offered by university faculty, Math CAMMP is the only camp that integrates teacher coursework with pedagogy with a summer camp for kids. It is an innovative and cutting edge instructional program that works for both students and teachers.

LESSONS FOR TEACHERS

Teachers who participate in Math CAMMP gain valuable skills and strategies that help them back in their classroom. The CAMMP for students helps the teachers overcome two significant impediments noted earlier: over-reliance on textbooks for teaching strategies (Willoughby, 1990) and teaching the same way they were taught as elementary students (Bauersfeld, 1998; Désautels, 2000). Teachers learn, practice, use, and receive feedback on:

- How to make math fun and educational for students;
- How to motivate students to achieve high goals;
- Incorporating problem solving as a highly motivating context for teaching math;
- How manipulatives speed up rather than slow down learning;
- How the same manipulatives are useful across a range of content (e.g., Cuisenaire rods for operations on whole numbers, fractions, and decimals);
- Strategies for checking answers that motivate students to check their own solutions instead of asking "Is this right?"
- How manipulatives can be used to differentiate instruction;
- How to develop a hierarchy of instructional techniques;
- How arithmetic and algebraic operations are related as actions on different objects;
- How to plan instruction that incorporates all the elements of CAMMP;
- How to incorporate continuous assessment;
- How to group students for instruction;

Math CAMMP is taught only during summer session, when students are available as "campers." This is consistently the first course to fill up, and its enrollment supports two professors as co-teachers. Teachers enroll in Math CAMMP in part because of the direct experience with students and in part because it is unlike any other graduate course they have ever taken. And the students continue to make Math CAMMP the most popular nonsports camp on campus.

REFERENCES

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*, 449-466.

Bauersfeld, H. (1998). Remarks on the education of elementary teachers. In M. Larochelle, N. Bednarz, & J. Garrison (Eds.), *Constructivism and education* (pp. 213-232). New York: Cambridge University Press.

Désautels, J. (2000). Science teacher preparation: An attempt at breaking the reproduction cycle of the traditional model of teaching. In L. P. Steffe & P. W. Thompson (Eds.), *Radical Constructivism in Action: Building on the pioneering work of Ernst von Glasersfeld* (pp. 195-212). New York: Routledge.

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht: Kluwer Academic Publishers.

Graeber, A., Tirosh, D., & Glover, R. (1989). Preservice teachers' misconceptions in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education, 20*, 95-102.

Green, M., Piel, J. A., & Flowers, C. (2008). Reversing education majors' arithmetic misconceptions with short-term instruction using manipulatives. *Journal of Educational Research, 101*, 234-242.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Piaget, J. (1974). *To understand is to invent*. New York: Viking Press.

Piel, J. A., & Green, M. (2010). *Constructing number sense in the elementary and middle grades classroom*. Dubuque, IA: Kendall Hunt.

Piel, J. A., & Green, M. (1994). De-mystifying division: The convergence of quantitative and referential meaning. *Focus on Learning Problems in Mathematics, 16*(1), 44-50.

Willoughby, S. S. (1990). *Mathematics education in a changing world.* Alexandria, VA: Association for Supervision and Curriculum Development.

Appendix

Table 1 Action Language for Four Arithmetic Operations

Operation	Action Language	Traditional Language
Addition	Joined with	Plus
Subtraction	Take away	Minus
Multiplication	Sets of	Times
Division	Grouped into	Divided by or goes into