Al-Jo’anee Company: support department cost allocations with matrices to improve decision making

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ABSTRACT

The direct, step-down and reciprocal methods are commonly used for allocating costs of support departments. While each method has benefits and drawbacks, the direct and step-down methods are used often because of their simplicity in allocating costs. In contrast, the preferred reciprocal method is difficult to adopt as simultaneous equations are needed for solving reciprocated costs of each support department. The Al-Jo’anee Company presents a matrix approach to model support services among departments, to solve for reciprocated costs of support departments, and to allocate the reciprocated costs to five support departments.

Keywords: matrix-based cost allocations, reciprocal method
INTRODUCTION

Support department costs are allocated to operating departments that generate revenues for the organization and create the need for support services. Hence, cost allocations are used to provide information for economic decisions, to motivate managers and other employees, to justify costs or compute reimbursements, and to measure income and assets for financial reporting (Horngren et al., 2006). As the number of support departments and their costs increase, the allocation of support department costs becomes increasingly important. While there is a preferred method of allocating support department costs, businesses have chosen to adopt simple methods of allocating costs even when they have drawbacks.

There are three methods of allocating support department costs: the direct, step-down and reciprocal. The key differences among the methods are the assumptions as to how services provided by one support department are allocated to other support departments. The direct method ignores services provided by one support department to another; hence, the direct method allocates support department costs only to operating departments. In contrast to the direct method, the reciprocal method captures all of the support departments’ services to other support departments. Hence, the reciprocal method allocates not only a support department’s own costs but costs allocated to it from other support departments. The step-down method represents a compromise between the direct and reciprocal methods in that it recognizes some, but not all of the services of support departments.

COST ALLOCATION METHODS

The direct method is the simplest of the three cost allocation methods. It ignores reciprocal or interdepartmental services among the support departments. The direct method allocates all costs of service departments directly to operating departments; hence, even if a service department provides a large amount of service to another service department, no allocations are made between the two departments.

The step-down method allocates costs of a service department to production departments and to some service departments. This method allows for only partial recognition of the services rendered by support departments. It is called the step-down method because a service department that has been allocated (closed) can receive no cost allocations back to it from the remaining unclosed support departments. Hence, the order of closing support departments will result in different allocations of support department costs to operating departments (Horngren et al., 2000).

The reciprocal allocation method is conceptually the best of the three methods because it recognizes all services among support departments. This method defines reciprocated costs of a support department as its own cost plus costs allocated to it from other support departments. It is the reciprocated costs of a support department that are allocated to all other departments based on their use of services.

While the reciprocal method is preferred over the direct and step-down methods, its adoption has been hindered by the requirement to solve for reciprocated costs of service departments using an algebraic set of simultaneous equations. Textbook examples will limit the number of support departments to three so students will not have much difficulty solving for the reciprocated costs of each support department. However, in many organizations the number of
support departments often exceeds three, and accounting graduates will have difficulty using the reciprocal method without another approach to solving simultaneous equations.

Spreadsheets have the capability to solve simultaneous equations with the use of matrices. The following case presents the matrix functions needed to model the reciprocal relationships among many service and operating departments, to solve for reciprocated costs of the service departments, and to allocate the reciprocated costs of a service department to other departments.

**AL-JO’ANEE COMPANY**

**Background**

Al-Jo’anee Company manufactures air-conditioning equipment in the Middle East. The demand for its products is increasing very quickly as more people are able to afford air conditioning. Hence, Al-Jo’anee has had to expand its product line to meet the new demand. As a consequence, the company is experiencing much higher costs in its support departments A, B, C, D and E.

In the past, the accounting department used the direct method for allocating support department costs to revenue generating departments. Managers of the profit centers have begun to complain that the allocations no longer reflect actual use of support services by revenue-generating and support departments. Top management also recognizes the need for more accurate allocations of support departments in an attempt to control increasing support costs.

The accountant at Al-Jo’anee refers to cost accounting textbooks and determines that the reciprocal method for allocating supporting department costs must be adopted. While it requires more data and work, the accountant knows that the reciprocal method should be acceptable to profit center managers. The accountant begins to gather data for implementing the reciprocal method.

**Departmental Data**

The cost for support departments A, B, C, D and E and two operating departments X and Y before any cost allocations are presented below. In addition, the percent of services provided by each support department to all other departments is presented. For example, Department A provides 0.04 and 0.40 of its services to Departments B and Y. The total of services for each support department is equal to 0.00, as the 1.00 sum of provided services is netted with the -1.00 allocated services.

<table>
<thead>
<tr>
<th></th>
<th>Dept A</th>
<th>Dept B</th>
<th>Dept C</th>
<th>Dept D</th>
<th>Dept E</th>
<th>Dept X</th>
<th>Dept Y</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs:</td>
<td>1,000,000</td>
<td>800,000</td>
<td>600,000</td>
<td>400,000</td>
<td>200,000</td>
<td>1,200,000</td>
<td>1,800,000</td>
<td>6,000,000</td>
</tr>
<tr>
<td>Services:</td>
<td>-1.00</td>
<td>0.04</td>
<td>0.08</td>
<td>0.09</td>
<td>0.07</td>
<td>0.32</td>
<td>0.40</td>
<td>0.00</td>
</tr>
<tr>
<td>Dept A</td>
<td>0.06</td>
<td>-1.00</td>
<td>0.09</td>
<td>0.07</td>
<td>0.08</td>
<td>0.20</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td>Dept B</td>
<td>0.05</td>
<td>0.06</td>
<td>-1.00</td>
<td>0.06</td>
<td>0.09</td>
<td>0.50</td>
<td>0.24</td>
<td>0.00</td>
</tr>
<tr>
<td>Dept C</td>
<td>0.07</td>
<td>0.05</td>
<td>0.05</td>
<td>-1.00</td>
<td>0.08</td>
<td>0.25</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td>Dept D</td>
<td>0.03</td>
<td>0.04</td>
<td>0.07</td>
<td>0.05</td>
<td>-1.00</td>
<td>0.41</td>
<td>0.40</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Reciprocal Method Simultaneous Equations

In the following algebraic expressions, A, B, C, D and E represent the reciprocated costs for the support departments of Al-Jo’anee Company. For Department A below, the reciprocated cost A is equal to its own costs of $1,000,000 and 0.06 of Department B’s reciprocated cost, 0.05 of Department C, 0.07 of Department D, and 0.03 of Department E. The equation can then be placed in a format more suited for matrix multiplication. Similarly, the formulas for reciprocated costs of the five departments are listed below as a set of simultaneous equations.

\[
\begin{align*}
\text{Department A:} & \quad +1.00A = 1000000 = +06B + 0.05C + 0.07D + 0.03E \\
\text{Department A:} & \quad +1.00A - 0.06B - 0.05C - 0.07D - 0.03E = 1000000 \\
\text{Department B:} & \quad -0.04A + 1.00B - 0.06C - 0.05D - 0.04E = 800000 \\
\text{Department C:} & \quad -0.08A - 0.09B + 1.00C - 0.05D - 0.07E = 600000 \\
\text{Department D:} & \quad -0.09A - 0.07B - 0.06C + 1.00D - 0.05E = 400000 \\
\text{Department E:} & \quad -0.07A - 0.08B - 0.09C - 0.08D + 1.00E = 200000
\end{align*}
\]

Matrix For Simultaneous Equations

An equivalent matrix equation for the formulas above is \([L] \times [M] = [N]\) as shown below. The \([L]\) 5x5 matrix is from the simultaneous equations above, which is the negative transposition of departmental services provided among the support departments. The \([M]\) 5x1 matrix is the reciprocated cost variables A, B, C, D and E. The \([N]\) 5x1 matrix is the individual costs of the five departments. Each value within a matrix is identified by specifying the matrix and its row and column. For example, \((L_{1,3})\) is equal to -0.05 as it is found in the L matrix at row 1 and column 3. An array of numbers is noted as \((L_{1,1}:L_{5,5})\), which is equivalent to the L matrix.

\[
\begin{array}{c}
[L] \\
+1.00 & -0.06 & -0.05 & -0.07 & -0.03 \\
-0.04 & +1.00 & -0.06 & -0.05 & -0.04 \\
-0.08 & -0.09 & +1.00 & -0.05 & -0.07 \\
-0.09 & -0.07 & -0.06 & +1.00 & -0.05 \\
-0.07 & -0.08 & -0.09 & -0.08 & +1.00 \\
\end{array}
\times
\begin{array}{c}
[M] \\
A \\
B \\
C \\
D \\
E \\
\end{array}
=\begin{array}{c}
[N] \\
1000000 \\
800000 \\
600000 \\
400000 \\
200000 \\
\end{array}
\]

Solving For Reciprocated Costs

The reciprocated cost for each department is computed by multiplying both sides of the matrix equation with the \([L^{-1}]\) inverse of matrix \([L]\), such that \([L^{-1}] \times [L] = [I]\) identity matrix and \([I] \times [M] = [M]\).

Given: \([L] \times [M] = [N]\)

\([L^{-1}] \times [L] \times [M] = [L^{-1}] \times [N]\)

\([I] \times [M] = [L^{-1}] \times [N]\)

\([M] = [L^{-1}] \times [N]\)

After selecting a 5x1 range for the \([M]\) solution, the following Excel formula multiplies \([L^{-1}]\) with \([N]\) to solve for \([M]\) reciprocated costs of each department. Press the Ctrl + Shift t + Enter keys together to generate \([M]\).

Excel formula: = mmult(minverse([L]),[N]) = mmult(minverse(L_{1,1}:L_{5,5}),N_{11}:N_{5,1})
Reciprocated Costs Allocations

The allocation of reciprocated costs for each support department to all other departments is performed by multiplying two matrices. The $[D_M] \times [P] = [A]$ matrix multiplication is shown below, where the $5 \times 5$ $[D_M]$ matrix is the diagonal matrix of $[M]$, and the $5 \times 7$ $[P]$ matrix is the original service departmental data. The Excel formula for the matrix multiplication follows.

Excel formula: $= \text{mmult(minverse([D_M]),[P])}$

$$[D_M] = \begin{bmatrix} 1158867 \\ 948678 \\ 844296 \\ 645596 \\ 484649 \end{bmatrix}$$

$$[P] = \begin{bmatrix} -1.00 & 0.04 & 0.08 & 0.09 & 0.07 & 0.32 & 0.40 \\ 0.06 & -1.00 & 0.09 & 0.07 & 0.08 & 0.20 & 0.50 \\ 0.05 & -1.00 & 0.06 & 0.09 & 0.50 & 0.24 \\ 0.07 & 0.05 & 0.05 & -1.00 & 0.08 & 0.25 & 0.50 \\ 0.03 & 0.04 & 0.07 & 0.05 & -1.00 & 0.41 & 0.40 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 1158867 & 0.00 & 0.00 & 0.00 & -1.00 & 0.04 & 0.08 & 0.09 & 0.07 & 0.32 & 0.40 \\ 0.00 & 948678 & 0.00 & 0.00 & 0.06 & -1.00 & 0.09 & 0.07 & 0.08 & 0.20 & 0.50 \\ 0.00 & 0.00 & 844296 & 0.00 & 0.05 & -1.00 & 0.06 & 0.09 & 0.50 & 0.24 \\ 0.00 & 0.00 & 0.00 & 645596 & 0.07 & 0.05 & 0.05 & -1.00 & 0.08 & 0.25 & 0.50 \\ 0.00 & 0.00 & 0.00 & 0.00 & 484649 & 0.03 & 0.04 & 0.07 & 0.05 & -1.00 & 0.41 & 0.40 \end{bmatrix}$$

The resultant $5 \times 7$ $[A]$ matrix is the allocation of reciprocated costs of support departments to all other departments. The $[A]$ matrix is placed in the cost allocation table below. The cost allocation is complete as support Departments A, B, C, D and E have zero balances and the total cost of all the departments remain the same at $6,000,000.

The reciprocal cost allocation method has transferred all support department costs to the operating departments and at the same time recognized all services performed by support departments.

<table>
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<tr>
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<td>1,200,000</td>
<td>1,800,000</td>
<td>6,000,000</td>
</tr>
<tr>
<td>Services:</td>
<td>-1,158,867</td>
<td>46,355</td>
<td>92,709</td>
<td>104,298</td>
<td>81,121</td>
<td>370,837</td>
<td>463,547</td>
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<tr>
<td>Dept B</td>
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<td>-948,678</td>
<td>85,381</td>
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<td>189,736</td>
<td>474,339</td>
<td>0.00</td>
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<tr>
<td>Dept C</td>
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<td>-844,296</td>
<td>50,658</td>
<td>75,986</td>
<td>422,148</td>
<td>202,631</td>
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</tr>
<tr>
<td>Dept D</td>
<td>45,192</td>
<td>32,280</td>
<td>32,280</td>
<td>-645,596</td>
<td>51,648</td>
<td>161,399</td>
<td>322,797</td>
<td>0.00</td>
</tr>
<tr>
<td>Dept E</td>
<td>14,539</td>
<td>19,385</td>
<td>33,926</td>
<td>24,233</td>
<td>484,649</td>
<td>198,706</td>
<td>193,860</td>
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</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,542,826</td>
<td>3,457,174</td>
<td>6,000,000</td>
</tr>
</tbody>
</table>

CONCLUSION

The Al-Jo’anee Company case uses spreadsheet matrix function to allocate the costs of support departments to operating departments for the reciprocal method. The matrix-based approach facilitates the use of the preferred reciprocal method of allocating support costs to operating departments. Solving for reciprocated costs of many support departments no longer requires tedious and difficult computations by hand. Matrix functions for multiplication and
inverses of a matrix are useful tools for solving simultaneous equations. The allocation of reciprocated costs for each department to other support and operating departments is simplified with the matrix multiplication function.

Accounting faculty and practicing accountants can now emphasize the reciprocal method for allocating support department costs to operating departments because of facilitative matrix functions available on spreadsheets. The use of the reciprocal method will improve cost information in making key management decisions.

REFERENCES