A framework for comparing theories related to motivation in education

Jeff Irvine
Brock University

ABSTRACT

Motivation is a meta concept with well-researched theoretical constructs, such as expectancy-value and intrinsic-extrinsic, and encompasses a myriad of related theories such as self-efficacy, goal theory, theories of intelligence, choice theory, self-determination theory, and flow, among others. This study developed a framework that enables visual comparison of such theories, highlighting their similarities and clearly differentiating major attributes; for example, differences between goal theory and theories of intelligence are clearly delineated, while similarities between self-determination theory and choice theory are visually evident. The framework can also be applied to subject-specific theories related to motivation, such as mathematical wellbeing.

Keywords: expectancy-value, extrinsic motivation, intrinsic motivation, motivation

Motivation has a number of definitions, ranging from "an individual's desire to act in particular ways" (Walter & Hart, 2009, p. 163) and "reasons individuals have for behaving in a given manner in a given situation" (Middleton & Spanias, 1999, p. 66) to more complex (and thus more informative) interpretations such as "a potential to direct behaviour that is built into the system that controls emotion," a potential that "may be manifested in cognition, emotion and/or behaviour" (Hannula, 2006, p. 166). Motivation is a meta concept that subsumes a number of related concepts such as engagement, persistence, interest, self-efficacy, and self concept. Because it is a meta concept, motivation involves a wide array of theoretical constructs—such as expectancy-value or intrinsic-extrinsic—and many related theories, including self-efficacy, goal theory, theories of intelligence, choice theory, self-determination theory, and flow, among others. This paper proposes a framework that enables visual comparison of such theories, highlighting their similarities and clearly differentiating their major attributes.

Foundations of the Framework

The proposed framework identifies two of the major theories or motivational constructs against which other theories can be compared: expectancy-value and intrinsic-extrinsic motivation. Consideration of alternative foundational theories can be found later in this paper.

Expectancy-Value Theory

Expectancy-value theory posits that students' choice of tasks, persistence, and achievement depends on two factors: students' beliefs about their probability of success and the value they place on the task (Eccles, 1994, 2005, 2009; Eccles & Wigfield, 1995, 2002; Wigfield & Eccles, 2000). Students choose a task based on degree of difficulty and the cost associated with that choice (Eccles & Wigfield, 2002; Eccles, Wigfield, & Midgley, 1993; Eccles, Wigfield, & Schiefele, 1998). We thus can see the interrelationship between expectancy-value theory and self-efficacy; the students' beliefs about their own ability to accomplish a given task will influence whether they choose to engage in the task. Ball, Huang, Cotton, Rikard, and Colemn (2016) point out that while expectancy and self-efficacy are different theoretical constructs, it is often often difficult to separate and load them onto the same factors in research studies.

Eccles and Wigfield (2002) propose four dimensions to task value: attainment value, the importance to the student of accomplishing the task; intrinsic value, related to the enjoyment of doing the task; utility value, which reflects the relevance of the task to present or future goals; and cost, which is the opportunity cost associated with choosing this task over another one, and includes estimates of effort required to complete the task and any negative emotions or anxiety caused by the task. A related construct is interest—the emotional aspect of value. It is difficult (perhaps impossible) to separate the emotional, affective portion of valuing from the importance portion (Koller, Baumert, & Schnabel, 2001). Elements of these task value dimensions can be seen in Marzano's treatment of motivation (Marzano, 1998; Marzano & Kendall, 2001, 2007), discussed later in this paper.

Much empirical research supports expectancy-value and the four dimensions of value (Guo, Marsh, Parker, Morin, & Seeshing Yeung, 2015). Penk and Schipolowski (2015) state that expectancy-value is the most common framework used to investigate test-taking motivation; their study revealed that both expectancy of success and value—as measured by interest, usefulness, and importance—significantly determined the effort expended on the test, which correlated strongly with achievement on the test. The expectancy-value model was used to measure factors influencing student choices in physical education (Pang, 2014); choice of technological design projects (Masson, Klop, & Osseweijer, 2016); decisions regarding science, technology, engineering and mathematics (STEM) course selections and careers (Ball et al., 2016; Lykkegaard & Ulriksen, 2016); and course design in teaching composition (Bailey, 2015). Gonzalez and Paoloni (2015) used an expectancy-value model to relate motivation in science classrooms to metacognitive strategies and autonomy support (c.f., self-determination theory), while Abraham and Barker (2015) used it to create a sustained enrolment model for physics (SEMP). Burak (2014) related expectancy-value and flow (Csikszentmihalyi, 1990, 1997) in studying the motivation of students taking instrumental music courses in Turkey. The placement of flow on this proposed framework is discussed later in this paper.

Flake, Barron, Hulleman, McCoach, and Welsh (2015) not only validated that cost is a dimension of value but also proposed a model for the cost dimension encompassing task effort, outside effort (consisting of time required for other tasks beyond the task of interest), loss of valued alternatives (opportunity cost), and emotional cost. Flake et al. found empirical support for their cost model in a study involving 123 university students.

There is also empirical evidence of the multiplicative impact of expectancy and value on motivation. Guo, Marsh, Morin, Parker, and Kaur (2015) used structural equation modeling (SEM) to demonstrate the importance of the multiplicative influence of expectancy and value in a longitudinal study of Australian high school students' postsecondary choices. They found a differential impact of motivation for STEM career choices based on gender. In a second study, Guo, Marsh, Parker et al. (2015) again verified the multiplicative factor (expectancy x value) when investigating motivation in mathematics and relating motivation to student background variables and gender, using TIMSS data from Hong Kong.

Eccles (2009) has further developed expectancy-value by situating it in a social cognitive theory in which expectancy and value function as mediator variables within a comprehensive predictive model of student task and behavioral choices. The model consists of five columns, of which expectancy and value are column 4, and the tasks, activities, and behavioural choices are column 5. Column 1 comprises cultural and social factors and beliefs, as well as personal family characteristics such as gender, ethnic group, demographics, and aptitudes. In addition, an important factor in column 1 is previous personal experiences. Column 2 consists of the individual's perceptions, affective reactions, and memories of all of the factors in column 1. Column 3 recognizes the individual's emerging personal and collective identities, self-knowledge, and future goals. Column 4's "expectancy and value" then act as mediators between the influence of the first three columns and the tasks, activities, and behaviours chosen. A key factor is that the choices in column 5 then feed back to impact and become part of the previous personal experiences. In her paper, Eccles (2009) offers empirical support for her model as applied to identity and gender roles.

The expectancy-value axis does not represent a dichotomy but instead consists of two different constructs: expectancy and value. This axis is clearly not dichotomous, since a theory may posit high levels of expectancy as well as high levels of value. Such a situation would be represented on the framework by an elongated ellipse on the expectancy-value axis. A theory of motivation for which expectancy is a major attribute will be closer to the expectancy end of the axis; alternatively, a theory for which value is a major attribute will be closer to the value end of the axis. Theories for which expectancy and value are both emphasized will be close to the middle of the axis. Expectancy and value function as mediator variables (Baron & Kenny, 1986) in that they are part of the causal link between independent and dependent variables, with both direct and indirect effects (Bailey, 2015; Eccles, 2009; Penk & Schipolowski, 2015).

Thus, a theory situated on the expectancy-value axis indicates the degree to which the components of that theory function as mediator variables. A second dimension is provided by intrinsic-extrinsic motivation, discussed below.

Intrinsic-Extrinsic Motivation

Motivation can be parsed into two subconstructs, based on the reasons or goals that give rise to an action: "The most basic distinction is between *intrinsic motivation*, which refers to doing something because it is inherently interesting or enjoyable, and *extrinsic motivation*, which refers to doing something because it leads to a separable outcome" (Ryan & Deci, 2000a, p. 55). Extrinsic motivation is sometimes referred to as contingent rewards or incentives; that is, the reward is contingent on successfully performing a task (Bénabou & Tirole, 2003). The school system, with its emphasis on grades and external confirmation by teachers, is a common example of an extrinsically motivated system. This is unfortunate, because research by Ryan, Deci, and others shows that intrinsic rather than extrinsic motivation is central to learning and to creating lifelong learners (Ryan & Deci, 2000a, 2000b; Spinath & Steinmayr, 2012)—and that lifelong learning is a major goal of education (Sungur, 2007). Intrinsically motivated students demonstrated increased time on task, persistence when faced with failure, choice of more difficult tasks, more elaborate monitoring of comprehension, greater creativity, selection of deeper and more efficient strategies, and more risk taking (Middleton & Spanias, 1999). Alternatively, extrinsic motivation is negatively correlated with lifelong learning (Lin & McKeachnie, 1999). In addition, there is a significant body of evidence showing that extrinsic rewards actually decrease intrinsic motivation (Bénabou & Tirole, 2003; Deci, Koestner, & Ryan, 2001; Middleton & Spanias, 1999; Ryan & Deci, 2000a). The implication of providing an extrinsic reward is that the activity is not inherently valuable in itself, thus reducing intrinsic motivation to participate.

The conflict between extrinsic and intrinsic motivation has unfortunate implications for the education system. Studies by Middleton and Spanias (1999) and Spinath and Steinmayr (2012) found that most children begin school with a high level of intrinsic motivation which begins to decline immediately, bottoms out around age 16, and thereafter remains stable at this lower level. Ryan and Deci (2000a, 2000b) argue that the decline in motivation can be attributed to school tasks that are not inherently interesting or enjoyable, and Middleton and Spanias (1999) claim that this often applies to mathematics tasks. A major finding of the research is that intrinsic motivation can be influenced by teacher behaviours (Middleton, 1995; Middleton & Spanias, 1999). Ryan and Deci (2000a) found that positive and negative feedback enhances and diminishes intrinsic motivation, respectively. Middleton (1995) found that high school teachers' focused more on content than on student motivation, while Middleton and Spinath (1999) found that students' intrinsic motivation can be affected by careful design of instructional activities.

Ryan and Deci (2000b, 2006) have demonstrated that extrinsic motivation is a continuous variable, measured by the degree of *internalization* that the learner attaches to the extrinsic motivator. For example, if the student is not driven by a desire to achieve high marks, extrinsic rewards are a very low motivational factor, and can even be demotivating. Alternatively, if the desire for high marks is congruent with a student's personal goals, such as demonstrating competence, or gaining entrance to a specialized program, the external motivator is said to be highly internalized into the student's motivational system and is relatively close to acting as an intrinsic motivating factor. In the same way if the student is seeking certification or credentialing in a skill area, this extrinsic factor can be very motivating. However, once the certification is obtained, there is little motivation to continue learning in that area, and the extrinsic motivation does not contribute to lifelong learning.

Intrinsic-extrinsic motivation provides a second continuous dimension against which to view other theories of motivation—a moderator dimension (Baron & Kenny, 1986) that indicates the degree to which behaviour is influenced. By placing intrinsic-extrinsic motivation on one axis and expectancy-value theory on a perpendicular axis (Figure 1), theories related to motivation can be mapped across four quadrants, identifying the major focus or foci of the theory (Figure 2). Such an approach provides a single lens to recognize similarities and differences among the various theories, enabling a visual comparison of the major attributes of each theory. While this approach constitutes only one possible lens for viewing the various theories (sometimes competing and sometimes overlapping), it enables a coherent method for comparing and contrasting the different theories.

By placing other theories related to motivation on this framework, it is possible to compare and contrast each theory's foci and to emphasize theories' salient features. The following section discusses the utility of the framework in relation to self-efficacy, achievement goal theory, theories of intelligence, choice theory, self determination theory, Robert Marzano's treatment of motivation, and applies the framework to a content-specific theory—mathematical wellbeing.

Self-Efficacy

Self-efficacy (Bandura, 1986, 1997; Pajares, 1997; Pajares & Miller, 1994) refers to individuals' judgment about whether they are capable of accomplishing a task. In mathematics, students' perceptions about their mathematical abilities are related to their intrinsic motivation (Middleton & Spanias, 1999). Changes in self-efficacy can result in major changes in achievement. S. Ross's (2008) study of PISA 2003 mathematics data found that a one-unit increase in self-efficacy resulted in a 32-unit increase in achievement; she discovered also that no other motivational variable (intrinsic motivation, goal orientation, instrumental versus relational view of instruction) had as significant an impact on student achievement. Unfortunately, self-efficacy is very resilient and difficult to change (J. Ross, 2009), and it is also domain and task specific (Bandura, 1997; Shunk, 1991). Consequently, a student's self-efficacy will vary, sometimes dramatically, for different subjects and for different tasks within a subject; still, given the major potential impact that changes in self-efficacy can have on student achievement, it must play an important role in any study relating motivation and achievement. Self-efficacy has also been found to be positively related to effort and persistence (S. Ross, 2008).

Bandura (1997) identified four influencers of self-efficacy: performance accomplishments (mastery experiences), vicarious experiences, verbal (social) persuasion, and physiological (emotional) arousal. Kennedy and Smith's (2013) study of professional learning communities and teacher self-efficacy revealed a link between reflective practice and self-efficacy; although the study was done with teachers, it is not unreasonable to consider that student self-reflection and student self-efficacy are linked. Because self-efficacy is a difficult construct to measure, Kennedy and Smith (2013) list various instruments that were developed "in a further attempt to peel away the layers of the elusive efficacy construct" (p. 134). Also, because self-efficacy is resilient and requires long periods of time to change (J. Ross, 2009), many interventions focus on other motivational constructs even though changes in self-efficacy can have an important impact on student learning (S. Ross, 2008).

Meece, Wigfield, and Eccles (1990) found significant links between self-efficacy and expectancy-value constructs using SEM to examine math anxiety in middle school students. Meece et al. again confirmed that while expectancy and self-efficacy are different theoretical constructs, they are perceived by students as identical and function in identical ways to influence student motivation.

Figure 3 illustrates the location of self-efficacy theory on the four quadrant framework. Self-efficacy falls clearly in the expectancy-intrinsic quadrant. Although self-efficacy may be influenced by extrinsic factors such as recognition of past performance in the form of grades or feedback involving praise, these factors are not central to the very intrinsic nature of self-efficacy: the self-perception of one's capabilities. Self-efficacy does not involve any value judgment about the importance of the task or subject, so the value portion of the framework is not involved.

Achievement Goal Theory

This theory identifies two major goal orientations, often referred to as mastery goals and performance goals (Dweck, 1999; Dweck & Leggett, 1988; Pintrich & Shunk, 2002). A mastery goal (also referred to as a learning goal, task goal, or intrinsic goal) is one in which the student's aim is to gain knowledge or skills. A performance goal (also referred to as an ego goal, ability-focused goal, or extrinsic goal) is a competitive goal in which the aim is to look good compared to others (Pintrich, 2000, 2003; Pintrich, & de Groot, 1990; Pintrich, Roeser, & de Groot, 1994; S. Ross, 2008). The choice between mastery goals and performance goals rests on a number of factors, including feelings of self worth, theories of personal intelligence, fear of failure, and fear of looking "bad" in front of others (S. Ross, 2008).

Performance goals can be further subdivided into performance-approach and performance-avoidance goals, based on students' beliefs that they will do well, or conversely, on a fear of failure (Berger, 2009; Van Yperen, Blaga, & Postmes, 2014). Elliot, Murayama, and Pekrun (2013) propose further subdivisions of goals based on three potential orientations: task-based, related to the demands of the task; self-based, with an internal metric of the value of the activity; and other-based, with an external interpersonal metric. Each of these orientations is then assigned a valence of approach or avoidance. Thus, this model can be represented by a 3x2 matrix describing six goal orientations, each of which may vary depending on a given situation;

the goal orientation may lie in any of the six cells, and this position could be different in another situation, or even change during the performance of the task (Elliot et al., 2013).

Performance-approach goals are positively correlated to self-efficacy, task value, and use of cognitive and self-regulatory strategies (Shunk & Pajares, 2005). Performance-avoidance goals have not been found to be predictive of positive achievement (Elliott & Dweck, 1988; Elliott & Harackiewicz, 1996; Elliott & McGregor, 2001; Elliott & Thrash, 2001). Mastery goals are positively correlated with self-efficacy, task value, cognitive strategy use, and self-regulated learning (S. Ross, 2008). Ryan and Deci (2000a, 2000b) found that mastery goals are correlated with intrinsic motivation, whereas both performance approach and avoidance goals are correlated with extrinsic motivation. Thus, students with a high degree of intrinsic motivation tend to demonstrate mastery goals, and students with high levels of extrinsic motivation demonstrate performance goals. The converse is also true; students with a mastery goal orientation tended to have greater levels of intrinsic motivation, while students with a performance goal orientation tended to have lower levels of intrinsic motivation, and higher levels of extrinsic motivation (Spinath & Steinmayr, 2012). These findings are not surprising, given the direct relationship between having an internal metric (mastery goals, intrinsic motivation) or an external metric (performance goals, extrinsic motivation). Goal orientation was found to be correlated with self-efficacy beliefs, task enjoyment, and interest (Spinath & Steinmayr, 2012).

Goal orientation is a strong predictor of achievement. Students with a mastery goal orientation outperform students with a performance goal orientation (Middleton & Spanias, 1999). Middleton and Spanias (1999) found that students tend to adopt their teachers' goal orientations; therefore, if teachers demonstrate that they value mastery goals, this should impact students' goal orientations and thus increase students' intrinsic motivation. One way to accomplish this is to emphasize criterion- rather than norm-referenced assessments (Wolters & Daugherty, 2007).

Mastery goals fall in the value-intrinsic quadrant, whereas performance goals are located in the value-extrinsic quadrant (Figure 4). Mastery goals are internally motivated, although they can be influenced by teacher behaviours, as noted earlier. Performance goals are influenced by external factors, such as teachers' or parents' comments and attitudes towards students. The placement of performance goals in the expectancy-extrinsic quadrants is consistent with Ryan and Deci's (2000a, 2000b) findings, in that both performance approach and avoidance goals have an external locus. Placement of the two competing goal orientations (mastery versus performance) on the framework is axiomatic, based on internal versus external metrics.

Theories of Intelligence

There are two dominant theories of intelligence, the first being that intelligence is essentially fixed at birth and cannot be substantially changed, which is referred to as the entity or fixed view of intelligence. The second theory is that intelligence is malleable and can be improved through effort, a view referred to as effort, incremental, or growth (Dweck, 2006; Dweck & Leggett, 1988). A student's view of intelligence, his or her *mindset*, has significant implications for behaviour. If intelligence is fixed, then there is no point in making an effort to learn; the outcomes of learning are predetermined by fixed intelligence and will not change significantly no matter how much effort is exerted. Thus, there is no motivation to engage and no motivation to learn. On the other hand, if intelligence can be augmented by effort, students will not be limited by their assumptions about their own level of intelligence, and they will be motivated to make an effort, engage, and learn.

Fixed mindsets, which are prevalent in mathematics, can lead to the concept of *learned helplessness*, “a condition in which, because of lack of successes and the attribution of failure to lack of ability, individuals begin to view success as unattainable” (Middleton & Spanias, 1999, p. 71). Learned helplessness persists when educators place a high value on success and low value on effort, do not support an environment in which failure and struggle are seen as part of the learning process, and do not provide students with diverse learning styles or appropriate opportunities to put forth sustained effort (Middleton & Spanias, 1999).

There also is a differential gender impact for theories of intelligence. Middleton and Spanias (1999) found that girls “tend *not* to attribute their successes to ability but *do* tend to attribute their failures to lack of ability, exactly the attributional style that leads to failure” (p. 70). This effect is exacerbated by socialization stereotypes. The consistent pattern that develops is that females are socialized into viewing mathematics as a male domain and into perceiving themselves as being less able than males to do mathematics. Males tend to feel more confident in learning mathematics, are more convinced of the usefulness of mathematics, and in general identify more than females do with mathematics. Gender-role stereotyping does not solely affect females with low ability and motivation; even girls with high ability may perceive mathematics as a male domain, or they may defer to the “dominant male role” because of other social pressures whether or not they perceive mathematics as a male domain (Middleton & Spanias, 1999, p. 78).

There is a direct relationship between students’ applicable theory of intelligence and their goal orientation. Dweck (1999) stated that different theories of intelligence foster different goal orientations. Learners with a fixed mindset lean towards performance goals that are directed at demonstrating high competence or avoiding negative judgments of abilities by others; they prefer tasks and courses that are familiar, easy, and require little effort. Students with a growth mindset prefer learning goals that are oriented at increasing competence through the learning of something new and important; they prefer novel and challenging tasks that allow students to develop their abilities in unfamiliar domains. Therefore, students’ theory of intelligence has a direct impact both on their attitude to learning (Blackwell, Trzesniewski, & Dweck, 2007; McMahon, 2008) and their propensity to engage in the learning (Kennett & Keefer, 2006). This is especially true when students fail to achieve.

According to Dweck (1999), having a fixed mindset predisposes students to see failure as a direct indicator of their low ability, which is distressing and disengaging, whereas students with a growth mindset use failure as a cue to try new strategies and to try harder. Mangels, Butterfield, Lamb, Good, and Dweck (2006) used functional magnetic resonance imagery to verify the links between students’ intelligence frameworks and their goal orientation. Mangel et al.’s study used questionnaires to identify students as having either an entity or growth mindset. The subjects were then given batteries of general knowledge questions, while their brain activity was monitored using functional magnetic resonance imaging (fMRI). Feedback was given in two forms: simple right-or-wrong feedback and feedback for wrong answers that provided information on what the correct answers should be. The subjects were then retested on the same material. Based on brain scans, the students who subscribed to an entity view of intelligence ignored most of the feedback provided about wrong answers, and instead focused on the number of questions they had answered correctly; on the retest, they showed very little improvement. On the other hand, students with a growth mindset paid greater attention to the corrective feedback offered and scored significantly higher on the retest. While these results make sense, to see theory confirmed by neuroscience is extremely interesting and informative.

Growth mindset is located in the expectancy-intrinsic quadrant. Fixed mindset resides in the expectancy-extrinsic quadrant (Figure 5). While both mindsets could be considered intrinsic, because both are internal theories of intelligence, a fixed mindset is developed from interactions with external factors, such as the form of praise and the expectations expressed by others. A growth mindset can also be influenced by external factors such as the form of praise but a growth mindset orientation is largely intrinsic, based on students' perceived self-efficacy.

Flow

Csikszentmihalyi (1990, 1997) identifies flow as "The state in which people are so intensely involved in an activity that nothing else seems to matter; the experience itself is so enjoyable that people will do it even at great cost, for the sheer sake of doing it" (as cited in Bakker, 2005, p. 27). Bakker (2005) describes flow as "a state of consciousness where people become totally immersed in an activity, and enjoy it immensely" (p. 26). Bakker goes on to discuss multiple definitions of flow, most of which have three common characteristics: absorption, enjoyment, and intrinsic motivation. Absorption is a state of total concentration, where time flies and everything else around the person fades into the background. Work enjoyment is related to intrinsic motivation. Bakker (2005) describes intrinsic motivation as "the inherent pleasure and satisfaction in the work [that individuals] are involved in" (p. 28). Csikszentmihalyi (1990) identifies the *flow channel* as the tension between skills and challenge: when skills exceed challenge, the result is boredom; conversely, when challenge exceeds skill, the result is anxiety. Optimal conditions for flow occur when skills and challenge are similar.

Fullagar, Knight, and Sovern (2013) touch upon the same characteristics of flow (i.e., absorption, work enjoyment, intrinsic motivation) and go on to contrast the challenge-skill balance with task-specific anxiety, which occurs when challenge far exceeds skill. There also are echoes of Vygotsky's zone of proximal development (ZPD) in Csikszentmihalyi's flow channel. The ZPD identifies optimal educational situations as those in which challenge is slightly greater that skill, but achievable with effort.

Fullagar et al. (2013) discuss the importance of flow to learning as follows:

> Flow also seems to have important implications for the development of learning. When individuals develop the skill necessary to perform an activity, they also begin to master the challenges inherent in the activity. As skills are acquired, new challenges have to be identified so that the balance between challenges and capabilities can be maintained. This cycle increases motivation, enhances competence, fosters growth, and extends the individual's capacities. (p. 239)

Liljedahl (2014) in turn defines engagement as achieving flow. This appears to be a very high threshold for engagement. It seems clear that engagement can occur without the necessity of reaching flow. Csikszentmihalyi (1997) further refined the relationships between skills and engagement, resulting in eight potential conditions: low skill and low challenge result in apathy; moderate skill and low challenge produce boredom; high skill and low challenge produce feelings of relaxation; low skill and moderate challenge produces worry; low skill and high challenge result in anxiety; high skill and moderate challenge result in feelings of control; moderate skill and high challenge produce arousal; and high skill and high challenge produce flow. This characterization allows for a wider consideration of engagement, because the arousal and control octants are also likely to result in engagement. The high skills–high challenge condition for flow has been verified by numerous studies (Bakker, 2005; Csikszentmihalyi, 1997; Demerouti, 2006; Eisenberger, Jones, Stinglhamber, Shanock, & Randall, 2005; Salanova, Bakker, & Llorens, 2006). This condition has implications for educators—namely, that both high skill and high challenge are important to engagement and thus to achieving flow.

Bakker (2008) has constructed and validated an instrument for assessing flow—the WOrk-reLated Flow Inventory (WOLF)—that uses questions about the three primary characteristics of flow (absorption, work enjoyment, and intrinsic motivation). Bakker (2005) also examined the relationship between flow, task conditions, and available resources. He segmented resources into personal resources (including skill level, self-efficacy, and self-esteem) and job resources, such as performance feedback, autonomy, social support, and coaching. This generalizes the skills-challenge relationship to seeking balance between job demands (challenge) and available resources (of which skills is one aspect). Flow thus would be achieved more likely if there is a balance between job demands and available resources.

We see here implications for teaching. All the conditions identified by Bakker as available resources need to be addressed to improve the chance that flow will be achieved. If flow and engagement are perceived as related, then performance feedback, student autonomy or choice, social support through group or cooperative structures, and coaching all need to occur. In addition, student personal resources (such as self-efficacy, self-esteem, and skill development) must be explicitly addressed. If all these conditions are met, student engagement is more likely to occur and be sustained.

Flow is an entirely intrinsic construct. While flow lies mainly in the value-intrinsic quadrant, some elements of flow involve a high perceived self-efficacy. Thus in Figure 6, flow is illustrated as having a segment in the expectancy-intrinsic quadrant as well.

Choice Theory and Self-Determination Theory

Glasser's (1998) choice theory proposes that everyone act intentionally to satisfy five basic needs: survival, belonging, power, freedom, and fun. Therefore, students' actions in school can be interpreted by referencing the needs that they act upon intentionally to address. For example, providing students with a choice of activities addresses their need for freedom. Students' acting out may be recognition of their need for power. While not explicitly a theory of motivation, Glasser's theory allows educators to structure activities and environments to increase student motivation, by providing situations that support students in addressing the five basic needs.

Choice theory resides principally in the intrinsic quadrants. The need for power is located on the expectancy side of the axis, while choice, freedom, and fun are on the value side of the axis. Power is sometimes influenced by extrinsic considerations, as is choice. Thus the diagram for choice theory has a portion in the extrinsic quadrants as well. Survival straddles both axes, as illustrated in Figure 7.

Self-determination theory (Deci & Ryan, 1980, 1991; Deci, Vallerand, Pelletier, & Ryan, 1994; Ryan & Deci, 2000a, 2000b, 2006) is closely related to choice theory. Self-determination theory posits three needs that motivate students: autonomy, competence, and relatedness. Deci et al. (1994) propose that students act intentionally to address needs within these three dimensions, which has implications for student motivation. Ryan and Deci (2000a, 2000b, 2006) point out that there is a direct link between autonomy and intrinsic motivation: Intrinsic motivation requires autonomy in the form of free choice to participate in an activity, without requiring or desiring an external reward.

Hannula (2006) provides an example of the operationalization of self-determination theory in the mathematics classroom, by differentiating needs and goals based on their levels of specificity:

> In the context of mathematics education, a student might realize a need for competency as a goal to solve tasks fluently or, alternatively, as a goal to understand the topic taught. A social need might be realised as a goal to contribute significantly to collaborative project work and a need for autonomy as a goal to challenge the teacher's authority. (p. 167)

I find that the goal linked to autonomy in the above example is simplistic and quite negative. A preferred choice for such a goal could be choice in activities undertaken, thus increasing intrinsic motivation as well as addressing the autonomy need.

Self-determination theory is almost entirely on the intrinsic side of the axis, although some aspects of relatedness may be influenced by extrinsic factors. Competence lies on the expectancy side of the axis, while autonomy is on the value side of the axis, as shown in Figure 8.

Marzano's Treatment of Motivation

In Marzano's New Taxonomy (Marzano, 1998; Marzano & Kendall, 2001, 2007), motivation is considered part of the self system and encompasses three dimensions: examining importance, self-efficacy, and emotional response. Thus, motivation to engage in a task involves (a) perception that the task is important, (b) belief that the student possesses the ability to succeed at the task, and (c) a positive emotional response to the task (Marzano & Kendall, 2007).

This treatment of motivation has elements of both expectancy-value theory and self-efficacy. Marzano suggests that there will be repeated feedback loops to the self system as the student engages in the task. These feedback loops will involve verifying that the current task is still more important than possible alternative tasks, a re-evaluation of self-efficacy based on task progress to date, and reassessment of emotional response to the current task; however, Marzano leaves these feedback loops as implicit and not illustrated on his linear taxonomy.

Marzano's treatment of motivation located on the four quadrant framework is shown in Figure 9. The graph shows a balance between efficacy on the expectancy side of the axis, and value. Interest, an emotional response to the task, is one of the dimensions of value identified by Eccles and Wigfield (2002), as is importance. Both importance and interest are located on the value side of the axis.

Because Marzano's treatment of motivation addresses a balance between both dimensions of expectancy-value theory, and explicitly contains aspects of emotional response (interest), it constitutes a fulsome and operizational definition of motivation that can be explored experimentally.

Application of Four-Quadrant Framework to Content-Specific Theory Related to Motivation

The four-quadrant framework can also be used to explore content-specific theories related to motivation. One of these theories is Mathematical Wellbeing (MWB; Clarkson, 2013; Clarkson, Bishop, & Seah, 2010), which explicitly recognizes the role of emotion in examining motivation in mathematics.

Schoenfeld (1992) identifies five aspects of mathematical learning: the knowledge base, problem-solving strategies, monitoring and control, beliefs and affects, and practices. He acknowledges that beliefs and affects can influence mathematics learning in both positive and negative ways; for example, negative beliefs, and thus poor self-efficacy, have been shown to reduce problem-solving performance and general performance in mathematical tasks (Akin & Kurbanoglu, 2011; Amirali, 2010). Hattie (2009) determined that mathematical anxiety had an effect size of -0.12 with respect to achievement; when compared to the expected student gains effect size of 0.40, the result would be a decrease of over one-half a standard deviation, or more than 20 percentile points.

Emotions, attitudes, and beliefs can be ranked based on longevity of formation. McLeod (1992) ranks emotions as most immediate and lists examples such as joy or frustration in solving nonroutine problems to typify emotional response in mathematics. McLeod points out that even solving a straightforward problem in mathematics produces a small visceral thrill. Attitudes take longer to form and are considered an accumulation of numerous emotional responses over time. McLeod's examples of attitudes include enjoyment of problem solving, dislike of geometric proofs, and preferences for discovery learning. Beliefs are longest in formation and thus are most difficult to change; these include beliefs about mathematics, about self, about mathematics teaching, and about the social context of mathematics (McLeod, 1992). McLeod points out that attention to the affective dimension of mathematics is critical to students' success.

MWB (Clarkson, 2013; Clarkson, Bishop, & Seah, 2010) was developed by concatenating an emotion taxonomy to Bloom's cognitive and affective taxonomies. The five stages of MWB—awareness and acceptance of mathematical activity, positively responding to mathematical activity, valuing mathematical activity, having an integrated and conscious value structure for mathematics, and being independently competent and confident in mathematical activity—recognize that learning mathematics is an interrelated and intertwined structure of these three dimensions. Clarkson et al. (2010) recommend that activities be structured to support students at their current MWB stage and provide opportunities for them to move to higher stages, through consciously addressing cognitive, affective, and emotional needs and strategies.

MWB's five dimensions place it substantially in the Value-Intrinsic quadrant, with a portion in the Expectancy-Intrinsic quadrant reflecting the fifth dimension, independently competent and confident in mathematical activity. This is illustrated in Figure 10.

Discussion

The four-quadrant framework proposed in this paper facilitates a visual comparison of the salient features of theories related to motivation. Firstly, the framework allows interrogation of the principal features within a theory. For example, in goal theory the framework clearly delineates the contrast between mastery goals and performance goals. In theories of intelligence, the framework illustrates a clear distinction between growth mindset and fixed mindset. Further, the framework allows subtle attributes of theories to be emphasized; for example, in flow, which is clearly intrinsic and value oriented, the framework allows the implicit component of expectancy to be revealed.

The framework also helps draw comparisons across theories. The distinction between goal theory and theories of intelligence is an example of this comparison, in which the framework clearly illustrates the major differences in these two theories. The very strong similarities between choice theory and self-determination theory is another example of how the framework assists in illustrating these attributes.

I am also struck by the dominance of intrinsic over extrinsic in many theories related to motivation. This is even more striking considering the dominance of extrinsic rewards in current education systems. Motivational theories emphasize the intrinsic dimension where research has shown important gains can be made in positively impacting student motivation. A significant body of evidence suggests that motivation has a major role in student achievement (Hannula, 2006; Koller et al., 2001; Malmivuori, 2006; Middleton, 1995; Middleton & Spanias, 1999). In addition, there is a demonstrated reciprocal relationship between motivation and achievement (Koller et al., 2001; Middleton & Spanias, 1999), which is particularly important in mathematics. In a longitudinal study of 110,389 Canadian students in Grades 3, 6, and 9, Ontario's Education Quality and Accountability Office found that students who did not meet provincial standards in any of the three assessments had less positive attitudes about mathematics, had much less positive perceptions of their mathematics ability, liked mathematics much less after Grade 3, and were less likely to connect new mathematical concepts to previous knowledge (Shulman & Kozlow, 2014). Further, a *Toronto Star* report on the practice of streaming students in secondary schools indicated that "The problem is that student achievement often has more to do with motivation than innate intelligence" (Maharaj, 2014, para. 1). Thus, low achievement leads to low motivation, and low motivation leads to low achievement, in a debilitating spiral.

Other Possible Frameworks

The two foundational axes of expectancy-value and intrinsic-extrinsic were selected because they are theories that address motivation, broadly construed. In addition, as indicated previously, there is abundant research supporting both of these theories and they also provide a distinction between mediator variables (expectancy-value) and moderator variables (intrinsic-extrinsic).

A possible issue with this model is that as noted, expectancy and value are different constructs. Consideration was given to using a three-axis framework, with mutually perpendicular axes consisting of intrinsic-extrinsic, expectancy as a second axis, and value as a third axis. This leads to several interesting considerations. For instance, while intrinsic-extrinsic can be considered a continuum, should the value or expectancy axis terminate at zero or extend into negative valences? While conceptually value could certainly be negative (interpreted as the student gaining value by not engaging with the task), expectancy cannot fall below zero, as there can be no less than zero probability of success. However, negative expectancy could be interpreted as anxiety or discomfort. Mathematics anxiety is a common, well-studied phenomenon that acts to severely impede achievement. This could be interpreted as a negative expectancy, since the learner approaches the task with not only no expectation of success, the mere act of approaching the task generates depression and anxiety. A greater issue, however, is whether the three-axis model allows for useful comparisons of other theories related to motivation. During the writing of this paper, multiple model configurations were examined. The three-axis model did not allow for straightforward comparison of theories, and did not facilitate comparisons of various theories across the three dimensions of expectancy, value, and intrinsic-extrinsic. Thus, the original two-axis model was found to be superior.

Selecting two other theories related to motivation to serve as a foundation for a framework would not yield the comparable breadth and richness of results that can be seen using the framework discussed in this paper.

Many of the other theories of motivation that could be considered as fundamental frameworks postulate three or more dimensions; therefore, such theories do not easily allow a one-to-one mapping of other theories onto this structure. For example, choice theory consists of five factors, and no other theory identifies constructs such as survival as relevant to motivation.

Another consideration for an alternative framework is to construct a three-dimensional framework using the three dimensions of motivation proposed by Marzano; the three axes would consist of importance, self-efficacy, and interest (an emotional dimension). However, further examination of this framework shows it would often not yield useful comparisons since many theories related to motivation do not deal with an emotional dimension, and those that do address concepts such as interest frequently include this construct in the value dimension. One exception to this is MWB, which explicitly includes emotion in its formulation. Figure 11 is a representation of MWB using a three-dimensional Marzano framework. The five stages of MWB are represented by the ellipsoid: awareness and acceptance of mathematical activity (emotional response), positively responding to mathematical activity (emotional response), valuing mathematical activity (value), having an integrated and conscious value structure for mathematics (value), and independently competent and confident in mathematical activity (self-efficacy). The ellipsoid is broader on the value and emotional response dimensions, and less broad on the self-efficacy dimension, reflecting that MWB has fewer attributes concerned with self-efficacy than with value and emotional response. This representation illustrates that both positive and negative responses are possible for each dimension of motivation.

A second consideration for foundational theories is self-determination theory (SDT). The three fundamental dimensions of SDT—autonomy, competence, and relatedness—form a natural three-axis framework; however, this framework suffers from two major flaws. First, mapping other theories related to motivation onto this framework is difficult and imperfect at best. For example, the expectancy dimension of expectancy-value theory could be mapped onto competence, but the value dimension does not map well onto either autonomy or relatedness. Similarly, self-efficacy maps onto competence, but not onto the other two axes. The second difficulty with an SDT framework is that it fails to elucidate key features of other theories, and it is difficult to identify similarities and differences among the theories. Choice theory can be mapped onto such a framework but doing so fails to yield useful new information. Therefore, an SDT framework is not a useful framework for comparing theories related to motivation.

Summary

The utility of any representation lies in its ability to provide a lens for exposing new information or for yielding deeper understanding of a construct. By this measure, the representation in Figure 11, while interesting, is of limited usefulness. Nor does an SDT framework prove functional for this purpose. The two-dimensional framework using expectancy-value and intrinsic-extrinsic allows both intra-comparisons within a theory and inter-comparisons across theories. The visual identification of a theory’s salient features enables efficient comparisons and also exposes the role of the elements of a theory as mediator or moderator. The two-dimensional framework provides a straightforward method for exposing the critical elements of the myriad of theories related to motivation and for gaining a clearer understanding of what causes students to engage or not engage in tasks related to their learning. The value in this two-axis framework revolves around the thought and analysis required to place a theory related to motivation on the framework, resulting in a visual comparison of sometimes highly diverse theories.

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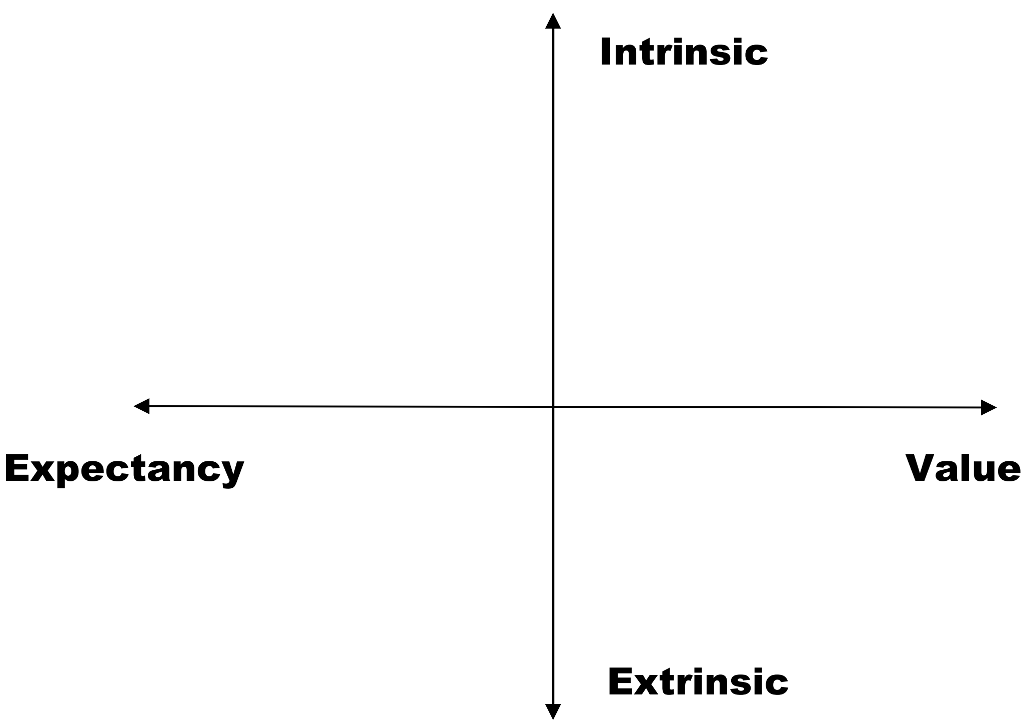
Figure 1. Axes for comparing theories of motivation



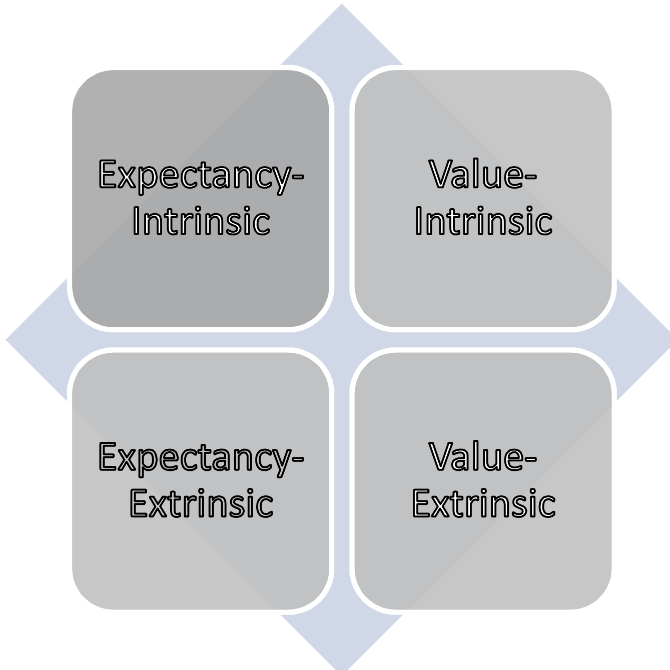


Figure 2. The four focus quadrants

Figure 3. Self-efficacy

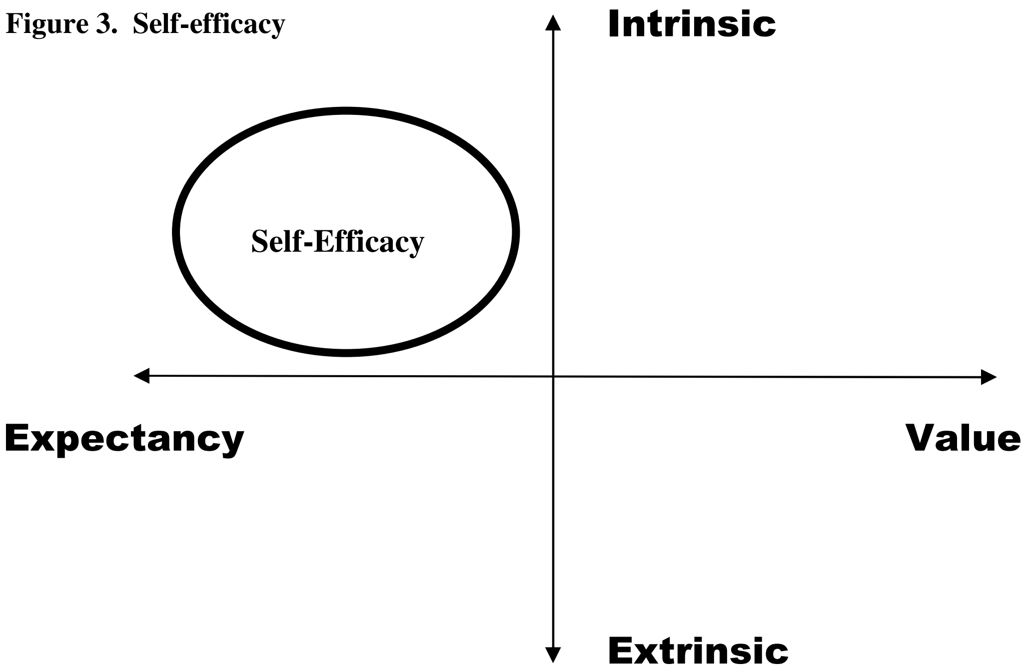


Figure 4. Achievement Goal Theory

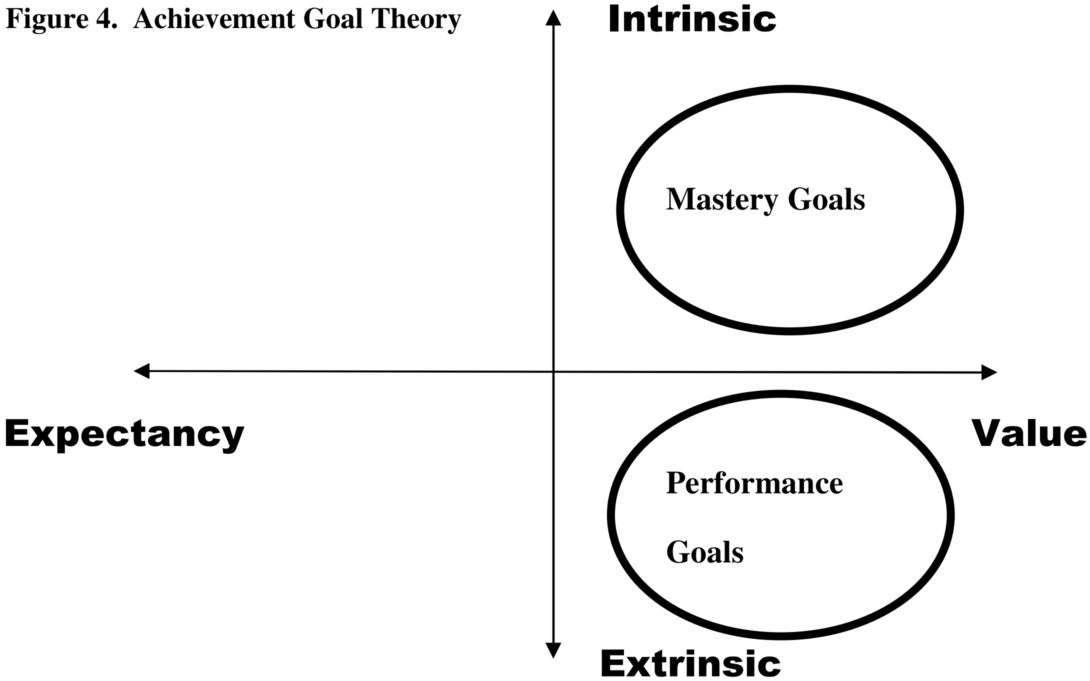


Figure 5. Theories of Intelligence

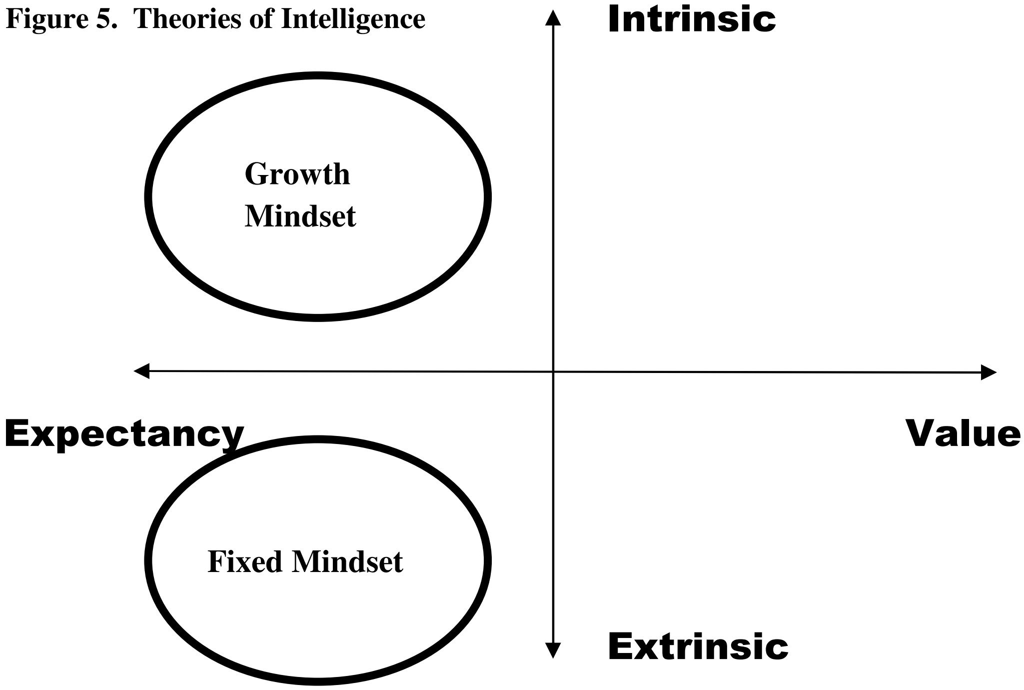


Figure 6. Flow

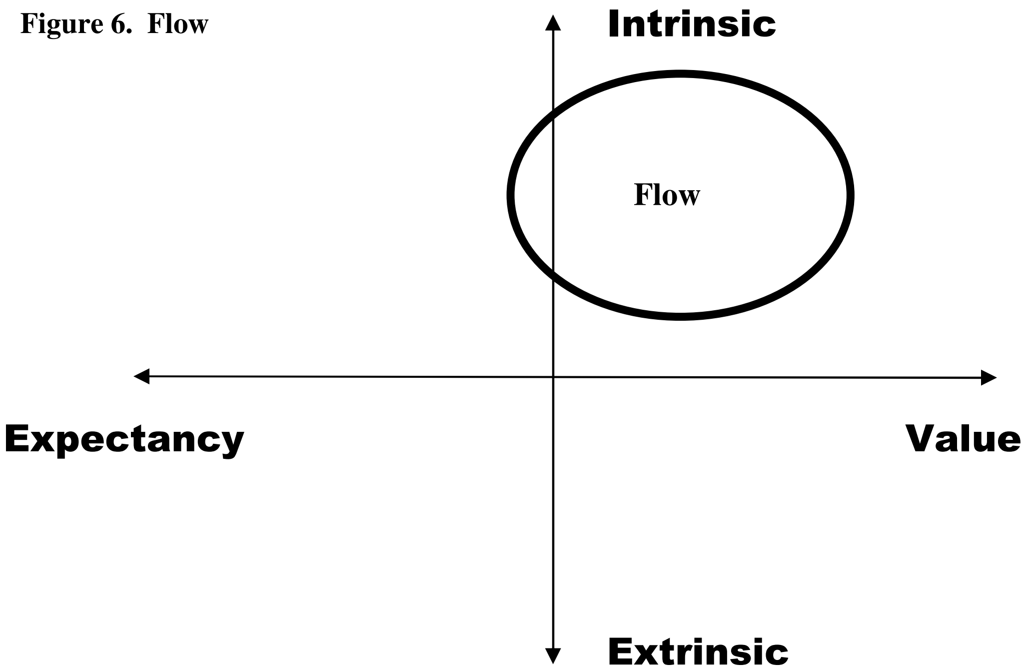


Figure 7. Choice Theory

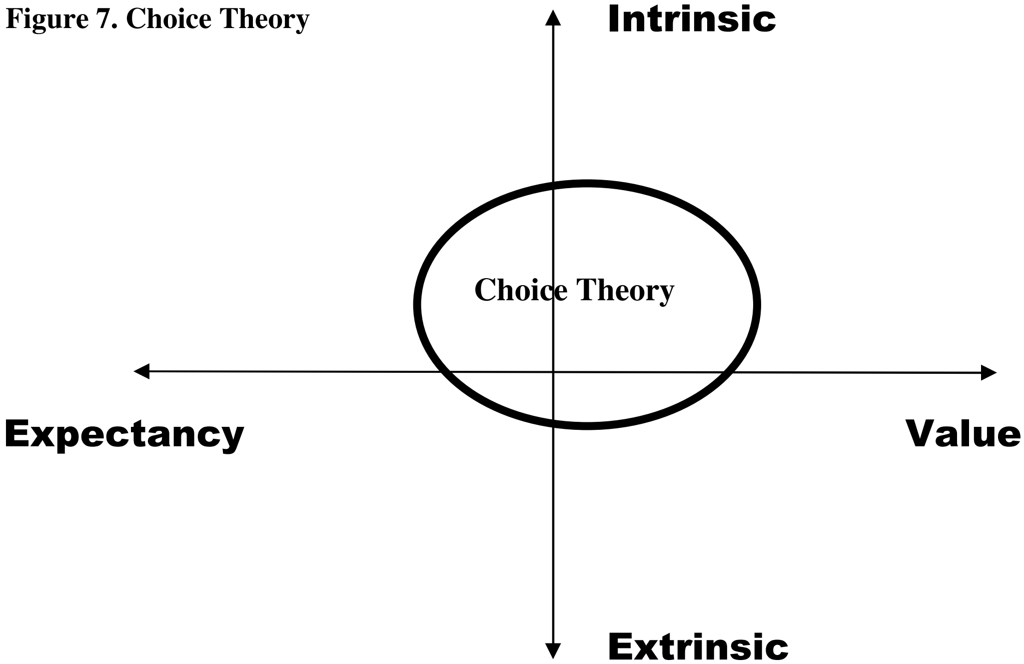


Figure 8. Self Determination Theory

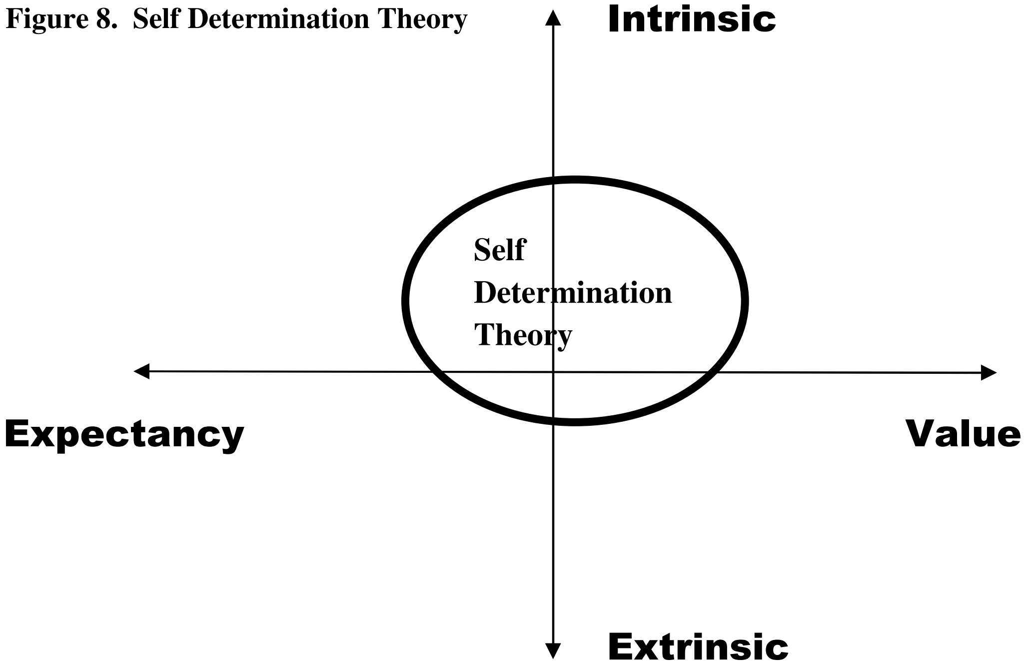


Figure 9. Marzano's treatment of motivation

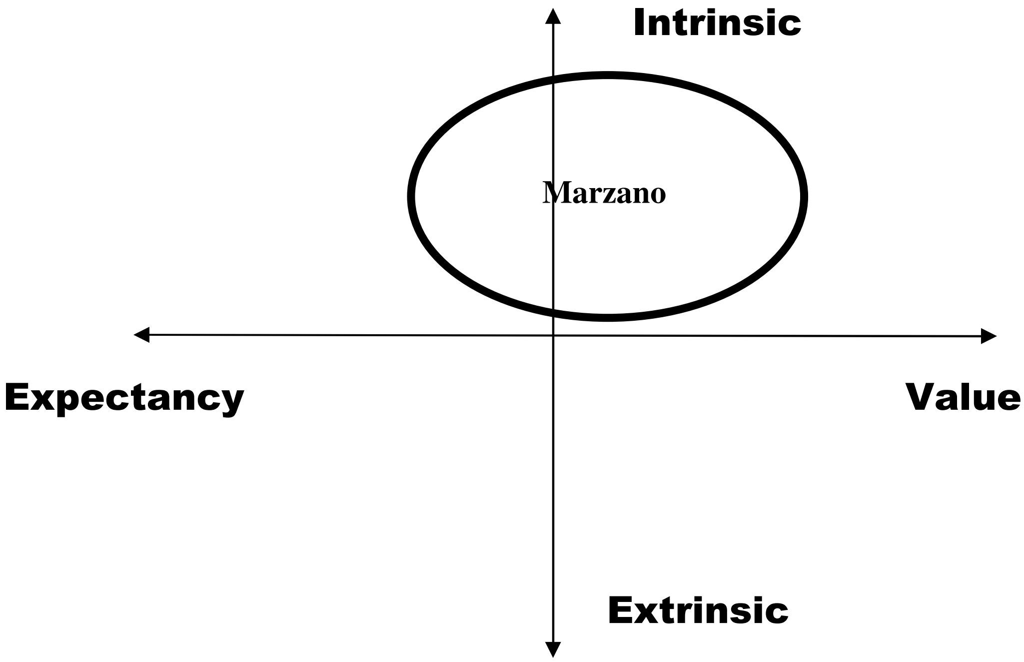
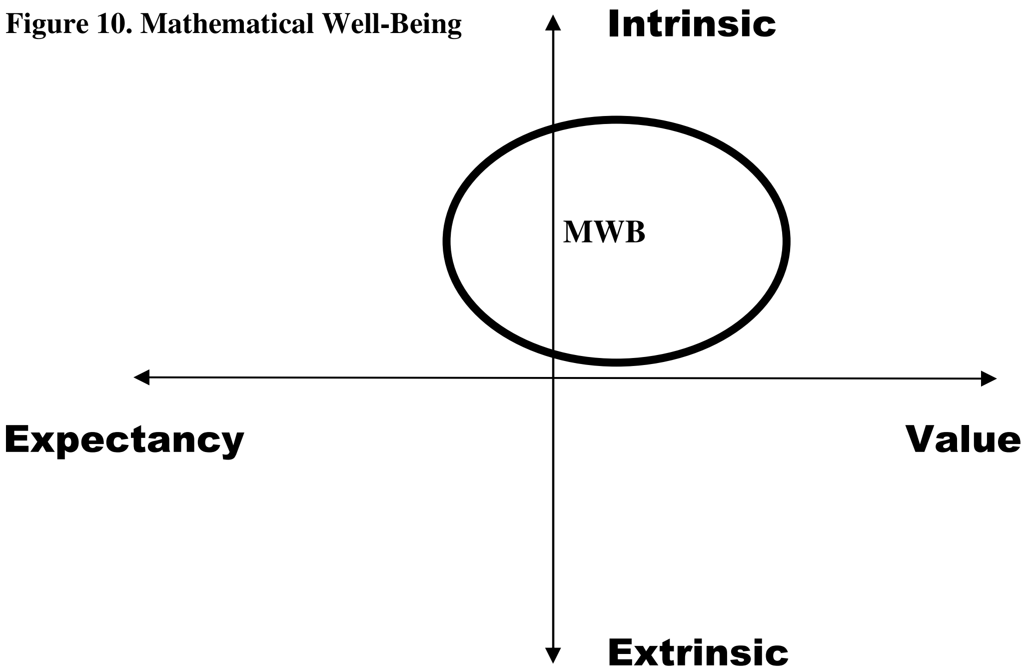


Figure 10. Mathematical Well-Being



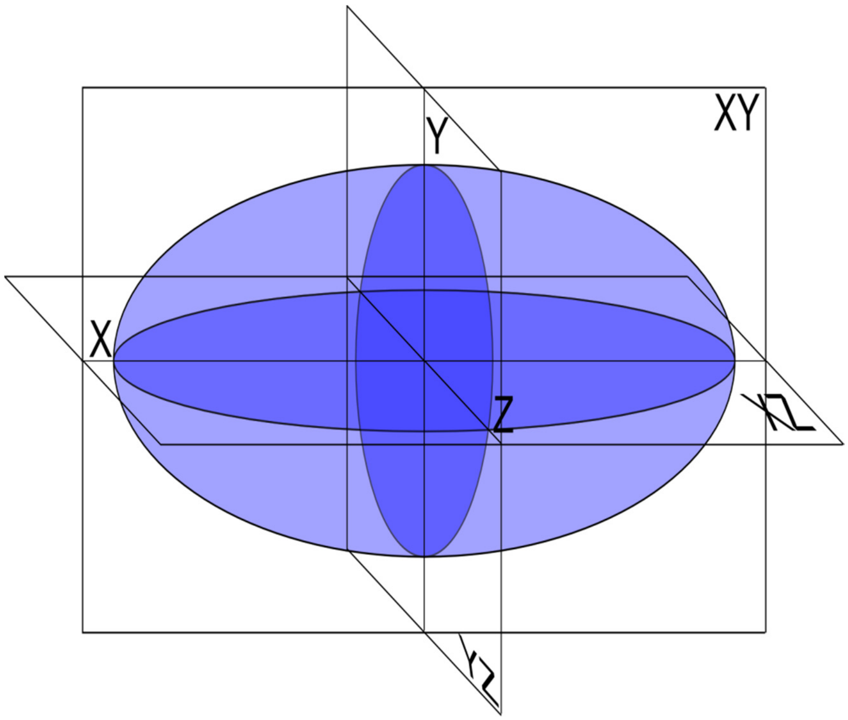


Figure 11. Illustration of Mathematical WellBeing on a three-dimensional framework based on Marzano's conception of motivation. The X-axis represents interest or emotional response; the Y- axis represents value, and the Z-axis is self-efficacy. All axes have positive and negative dimensions. Thus the XY plane is a representation of the interaction between emotional response and value; the YZ plane is value- self-efficacy; and the XZ plane is emotional response-self-efficacy.